

H C Walton

THE
YOUNG MEASURER'S
COMPLETE GUIDE;
OR A NEW AND
UNIVERSAL TREATISE
OF
MENSURATION,
BOTH WITH REGARD TO
THEORY AND PRACTICE.

CONTAINING:

I. A plain and easy introduction to decimal and duodecimal arithmetic, and the extraction of the square and cube roots.

II. A select number of geometrical problems useful in mensuration.

III. The method of measuring all kinds of superficies.

IV. The whole art of surveying or measuring land, explained in the most easy and familiar manner. Also the method of measuring heights and distances.

V. The method of measuring the works of bricklayers, joiners, carpenters, glaziers, painters, plasterers, and plumbers.

VI. The art of measuring solids, both regular and irregular.

VII. The art of gauging all kinds of vessels.

VIII. The practical and true methods of measuring timber, stone, &c.

IX. The manner of calculating tables for measuring timber, &c.

X. The best methods of taking the dimensions of timber, so as to avoid the gross errors attending the common practice.

XI. The art of measuring standing timber.

XII. The manner of finding the tonnage of ships very accurately.

XIII. The method of measuring sawyers work.

XIV. Of measuring cases, &c. sent by sea, and which pay freight by the solid foot.

XV. The method of measuring cord-wood, marble-pits, hay-ricks, statues, and other irregular bodies.

XVI. The construction of the plain scale, and its various uses in the practical branches of the mathematics.

The whole laid down in so easy and familiar a manner, as to render it not only useful for youths at school, but also necessary for artificers of every kind, and for such gentlemen as are desirous of surveying their lands, measuring their buildings, timber, &c.

The SECOND EDITION, with many Improvements.

By D. FENNING. *R*

Author of the British Youth's Instructor, of a New and Easy Guide to Practical Arithmetic, and the Royal English Dictionary, &c. &c. &c.

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E R R A T A.

Page 128, for *Book II*, read *Book III*. Page 257, for
Rule IV, read *Rule V*.



T H E
A U T H O R ' s
P R E F A C E.

THE several books I have published for the use of schools, and persons who have not enjoyed a mathematical education, having met with a kind reception, my friends, particularly those, who devote their whole attention to the instruction of youth, have often intreated me to compose a treatise on the Art of Measuring. This request I have accordingly complied with, and hope the work will be as kindly received, as my former publications; especially as the reader will meet with several particulars here, not to be found in any other book on the same subject.

Clearness and perspicuity have always regulated my conduct in compositions of this kind. I well know, that elegance and conciseness are esteemed by scholars, as the principal beauties in mathematical disquisitions; and that, in proportion, as these more or less abound, the book is said to be more or less valuable. But this treatise is intended to answer a very different purpose: It is designed to instruct those, who are wholly unacquainted with the mathematical sciences, in the art of mensuration; and in order to accomplish this valuable end, I have not scrupled to sacrifice both brevity and elegance. Those who have been employed in the instruction of youth, know, from painful experience, how difficult it is to make their pupils acquainted with any branch of learning: their instructions must be often repeated, and even the most minute particulars explained, before they can obtain the desired purpose.

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In the course of my teaching, I observed, that the best books on mensuration required too much attention for youth; that they abounded with difficulties they were unable to surmount; and that some parts of them could not be understood, unless the scholar had previously made a considerable progress in geometry and algebra, or had recourse to other writers. These difficulties continually occurring, I determined to draw up a manual for my own use, in order to remove these obstructions, and save myself the trouble of being perpetually engaged in explaining the same things.

In order to this, I have begun with a concise treatise of decimal arithmetic, the extraction of the square and cube roots, and duodecimal arithmetic, or what is commonly called cross multiplication. These fundamental rules I have endeavoured to lay down in the plainest manner; and I hope it will be found that I have not laboured in vain.

The geometrical problems, useful in practical mensuration, are next considered with all the clearness possible; and, I dare say, may be understood by a common capacity, without any other assistance.

The manner of measuring or finding the area of superficies follows: and every difficulty that might obstruct the learner's progress is entirely removed; though at the same time the reasons on which each rule is founded are given, where it was thought necessary.

These principles are next applied to practice. The whole art of surveying, or measuring land is explained, and, I hope, in such a manner, that every person may be able, with very little attention, to take the dimensions of a farm, or other parcel of land, plot it, and give the content in acres, roods, and perches.

The methods made use of by artificers, as bricklayers, carpenters, glaziers, joiners, painters, plasterers, &c. in measuring their works, are fully displayed by precepts and examples. I have here been very careful to shew the learner how he should proceed in taking the dimensions of these respective works, in
order

THE AUTHOR'S PREFACE. v

order to guard him against errors which might otherwise creep into his calculations.

The mensuration of solids is next treated of; and I have been careful to insert the method of measuring every species, necessary to be considered in a practical treatise: for with regard to those of a higher kind, whose properties depend upon principles of a more abstract nature, I have thought proper to omit them entirely, as they are of no use in actual mensuration.

From these principles the practical rules are deduced, and applied to those arts that are more immediately necessary to be known. The methods used in gauging all kinds of vessels are explained in an easy and familiar manner; and nothing omitted that is necessary to be known in that branch of the mathematics.

In the mensuration of timber, stone, &c. I have been very full; because it is an art of the utmost importance to be thoroughly known; the common methods being subject to such a variety of errors, that unless a person is well skilled in the fundamental principles, he is in danger of selling his timber at almost one fourth less than its real value.

I well know, that the common method of measuring by the girt, however false it may be in itself, is not to be eradicated by reason and demonstration: nor have I attempted it; but I have shewn the learner how he may take his dimensions, even on the common principles, in such a manner, as to avoid the errors that result from ignorance and confidence. I have also laid down an easy method for the construction of tables useful in measuring timber; so that any person may easily compute a set of tables for his own use, or examine those inserted in other books.

I have also explained the best methods for measuring standing timber: and likewise how to find the tonnage of ships, both by the common rules, and by a very accurate process.

vi THE AUTHOR'S PREFACE.

The manner of measuring sawyer's work is next fully explained; the errors attending the common practice pointed out, and a more accurate method substituted in its room.

To these I have added the methods pursued in finding the contents of goods sent by sea, which pay freight by the cubic foot; in measuring cord-wood, marle-pits, hay-ricks, statues, and other irregular bodies.

Such is the substance of the following treatise; how it may be received by the public must be left to time, the great discoverer of all events. I have assiduously laboured to render it a plain, easy, and useful introduction to the art of measuring; and if in endeavouring to execute this intention, I have sometimes overdone it, and used more words and figures than were absolutely necessary, I am sure the candid and ingenious will readily forgive me, as the fault flowed entirely from an earnest solicitude to render the work more useful to learners.

Errors, no doubt, may be found in this treatise; but I hope there are none that will impede the progress of the reader. And with regard to the typographical faults of the press, I presume they will be forgiven, as they are common to all books, especially those which treat of mathematical subjects.

23 OC 62

D. FENNING.

THE

THE
EDITOR'S
P R E F A C E.

THE immature death of the author preventing him from putting the last hand to this treatise, the papers were sent to me, with a request that I should revise the whole, and complete the parts that remained unfinished. Upon looking over the manuscript I found, that very few alterations were necessary, but that several of the chapters wanted considerable additions; and that some particulars were still wanting to render it a complete treatise of mensuration. I have therefore supplied these defects, and endeavoured to imitate the author's style and manner as near as possible. If I have been happy enough to succeed, I can venture to say, that the book will answer the most sanguine expectations of the public; for the rules given by the author are delivered in so plain and easy a manner, that a person even of the meanest capacity may easily comprehend them, and with very little trouble become an expert measurer.

It will also be of the utmost use in schools, as it will save the master the trouble of being perpetually engaged in explaining obscure passages, too often found in other authors; and the scholar the painful vexation of studying rules, he is not qualified to comprehend.

Nor are its uses confined to schools; private gentlemen and others, who are desirous of surveying their own estates, measuring their buildings, their timber, &c. will find this treatise a very useful companion; as it will remove every difficulty that might otherwise obstruct their design, and point out the best methods of proceeding in any particular they may desire to know.

Such youths as have not enjoyed the happiness of being instructed at school in this useful branch of learning,

viii THE EDITOR'S PREFACE.

learning, may, by the assistance of this treatise, and a little close application, obtain, by their own assiduity, what their situation in life had denied them. And even those, who have for some years been engaged in the practical branches of mensuration, will probably find several particulars in this work, deserving their attention; and which they may practise with credit to themselves, and advantage to their employers.

I must not conclude this preface without acknowledging, that the construction and use of the plain scale was added at the request of a very ingenious gentleman, who well deserves the thanks of the public, for the number of scholars he has brought up, many of whom are now some of the brightest ornaments of society.

ADVERTISEMENT

TO THE

SECOND EDITION.

THE Editor might justly be charged with ingratitude, did he not with pleasure take this opportunity of returning his thanks to the public, for the very favourable reception they have given to the first edition of this work. At the same time he begs leave to assure them, that he has endeavoured still further to merit their good opinion, by rendering this edition more complete than the former, partly by correcting the errors that had unavoidably escaped him, partly by inserting in it, page 97, &c. a more full and accurate method of measuring heights and distances, than that which was given in the first impression.

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THE INTRODUCTION.

CHAP. I. DIALOGUE I.

Between PHILO, a Master or Tutor, and DISCIPULUS, a Scholar; concerning The Rudiments of Decimal Arithmetic.

Discipulus. WILL you permit me, dear Sir, to ask you a favour? All my acquaintance are so pleased with the great progress Tyro and Tyrunculus have made in the most useful parts of learning, by your judicious instructions, that I could not be easy till I came to beg you will be kind enough to take me under your care, and make me acquainted with the art of measuring, which I have for some time desired to understand.

Philo. You are sincerely welcome, Discipulus. It gives me the greatest pleasure to see youths desirous of improving their minds, especially in arts so absolutely necessary for the happiness of society. But as I know not, Discipulus, whether you are acquainted with Decimal Arithmetic, the Extraction of the Square and Cube Roots, and the short methods of computation used in measuring superficies and solids; I should be glad to be informed, how far you have proceeded in Arithmetic, that I may lay the proper foundations of
B that

that art, before I proceed to explain its rules; for I hope you are already convinced, that no person can be properly said to understand any art, till he is acquainted with the principles upon which it is founded.

Disc. I am, Sir, indeed thoroughly persuaded of the truth of your observation; and have always thought, that the true reason why the generality of boys, when they come from school, are so ignorant, and so soon forget all they have learned there, is, their having never been thoroughly instructed in the fundamental principles. With regard to myself, I am entirely ignorant of Decimal Arithmetic, and the other rules you mentioned; and therefore you will begin with the rudiments of the art, and I hope, by attentively listening to all your instructions, to gain a thorough knowledge of the art of measuring, which I have so long wished to attain.

Philo. There is no doubt, my dear Discipulus, of your acquiring, in a short time, this useful branch of learning: it depends upon a few principles, easy to be understood, and extensive in their application. I shall therefore, without any farther preface, proceed to the rudiments of Decimal Arithmetic.

DIALOGUE II.

Of the Nature and Properties of Decimals, or Decimal Fractions.

Discipulus. **W**HAT do you mean by Decimals, or Decimal Fractions?

Philo. In order to answer your question, it will be necessary to premise, that any thing, considered as one, is called an unit, or integer: As one pound, one yard, one gallon, &c. are all units of their respective kinds. But, at the same time, each may be considered as composed of several lesser quantities; and consequently, these divisions, whatever they are, must be considered as parts of their respective units. I will endeavour to explain my meaning, by a very familiar example. You very well know that a yard is often divided into four quarters, and contains three feet, or thirty-six inches.

You

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You may therefore consider a yard, though properly an unit or integer, as composed of three feet, or thirty-six inches; or any other number of equal parts, into which it may be supposed to be divided.

Disc. I think I understand your meaning sufficiently. You observed, that a gallon, or any other thing, considered as a whole, is an integer. But I know that there are four quarts, or eight pints in a gallon; and therefore the gallon may be considered as composed of four quarts, or eight pints.

Philo. You are very right, Discipulus. I am glad you so easily comprehend my meaning; and, for your encouragement, let me add, that I will endeavour to make the whole as easy to you, as this leading principle; attention and a good memory in the scholar, being all that is necessary for attaining any art, when assisted with proper instructions.

Disc. You give me great encouragement, Sir, and I will sincerely endeavour to fix all your rules and precepts strongly in my memory. But though I very well comprehend, that any integer or unit may be considered as made up of different parts, yet I do not know what you mean by decimals, or decimal fractions.

Philo. You certainly do not; I am now going to explain, what is meant by decimal fractions. The parts, into which any integer or unit may be conceived to be divided, are called the fractions of that integer. Thus one inch is the one thirty-sixth part of a yard, because thirty-six inches make a yard, and is expressed in this manner $\frac{1}{36}$. That is, the number of parts into which the unit, or yard, is supposed to be divided, is placed below the line, and the one, or inch above it.

Disc. How are the two numbers that compose the above fraction to be distinguished, if they are called by the same name?

Philo. They have different names; that above the line being called the numerator, and that below it, the denominator:

Thus $\left\{ \begin{array}{l} \text{1 Numerator.} \\ \text{36 Denominator.} \end{array} \right.$

Disc. Is there any reason for giving these numbers different appellations?

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Philo. Yes. The number placed under the line is called the denominator, because it shews or denominates the number of parts, into which the unit or integer is divided; while the number above the line is termed the numerator, because it expresses the number of the same parts that are contained in the fraction.

Disc. I thank you, Sir; I understand you very well, as I suppose these names are general, and not confined to this particular fraction.

Philo. You are very right, Discipulus. They are general names; and I will now shew you how you may always know which is the numerator, and which is the denominator, by this general

RULE.

The denominator is always the divisor; the number of parts into which the integer is naturally divided, is the dividend; the number of the former contained in the latter, is the quotient, and the numerator is the remainder, after the division is finished.

I will illustrate this rule by an example. A yard is naturally divided into 36 inches; but suppose it were required to divide it into five parts: the work will stand thus:

$$\begin{array}{r} 5 \overline{) 36} \quad (7 \\ \underline{35} \\ .1 \end{array}$$

And the answer will be 7 inches and $\frac{1}{5}$ (or one-fifth) of an inch.

Disc. This is indeed very easy to be understood: but suppose there be no remainder, will there be no fraction?

Philo. There will not, my dear Discipulus. The quotient then will express the number of parts contained in each of those into which the integer is supposed to be divided. Thus suppose the yard were to be divided into six equal parts, the work will stand thus:

$$\begin{array}{r} 6 \overline{) 36} \quad (6 \\ \underline{36} \\ : \end{array}$$

And

INTRODUCTION. ▼

And as there is no remainder, there is no fraction; but the quotient shews, that 6 inches are contained in each of the six parts, into which the yard is supposed to be divided.

Disc. I think I thoroughly understand your instructions in this particular. If a gallon were supposed to be divided into 3 parts, each of these parts would contain 2 pints and two-thirds of a pint; because the divisor is 3, the dividend 8, and the remainder 2.

Philo. It gives me great pleasure that you so readily comprehend my meaning. But I must inform you, that these are called vulgar fractions, and are very different from decimals; but it was necessary first to explain the nature of fractions in general, in order that those of a particular kind may be the more easily understood.

Disc. Decimals are then only a particular kind of fractions. Are they very different in their nature, and in what does this difference consist?

Philo. They are very different from vulgar fractions, and managed with much greater ease and expedition: but the difference consists wholly in the denominators, which are always 10, 100, 1000, 10000, &c. that is, the integer or unit is supposed to be divided into 10 equal parts, and each of those parts into ten others, &c. The integer being thus divided into 10, 100, 1000, 10000, &c. equal parts, they become the denominators to decimal fractions.

Disc. This is very easy to be understood; but I should be glad to know why they are called Decimals, or decimal fractions.

Philo. They have their name from the Latin word *Decem*, which signifies *ten*, because the denominators of decimals increase in a ten-fold proportion.

Disc. I thank you Sir, for this explanation: the original import of the word tends greatly to fix the idea it conveys more firmly in the memory.

Philo. You have made a very just observation, and you will please me by often asking questions of this kind, as they might otherwise slip my memory, and I would not have you ignorant of any thing that can give you satisfaction, in the course of your studies.

DIALOGUE III.

Of Notation of Decimals.

Philo. **H**AVING, I hope, given you a proper idea of the nature of fractions in general, and the properties of decimals in particular, I shall proceed to the Notation of the latter.

Disc. I am greatly obliged to you for your care and attention to remove every difficulty; and will, in return, listen with the utmost attention to every part of your instructions.

Philo. As the denominators of decimal fractions are always known, it is unnecessary to set them down, and accordingly the numerators only are expressed; but they are separated from the whole numbers by a point or comma.

Thus: 5.4, is $5\frac{4}{10}$; 0.7, is $\frac{7}{10}$; 35.05, is $35\frac{5}{100}$, &c.

But perhaps you will conceive a better idea of these fractions from the following table, which shews the very foundation of decimals; and therefore deserves to be very attentively considered.

5	4	3	2	1	0	1	2	3	4	5	6	
						Unit's Place.	Parts of 10 or $\frac{1}{10}$.	Parts of a Hundred.	Parts of a Thousand.	Parts of 10 Thousand.	Parts of 100 Thousand.	Parts of a Million.
						Tens.	Hundreds.	Thousands.	Tens of Thousands.	Hundreds of Thousands.		&c.

Disc.

Disc. I will endeavour to understand this table, if you will be kind enough to explain it.

Philo. That I will most readily. This table is intended to shew, That, as in the whole numbers, every degree from the unit's place increases towards the left hand, by a ten-fold proportion: so in decimal parts, every degree is decreased towards the right hand by the same proportion, namely by tens.

Disc. This appears very plain: and I think I perceive the reason why you called 35.05, where the cypher follows the point of separation, $35.\overset{0}{\underset{5}{100}}$. because the 5 is by the cypher removed into the decimal parts of a hundred.

Philo. You are extremely right; and here you see the reason why cyphers, which placed on the left side of numbers are of no signification, are very different when placed on the left side of decimals. And on the contrary, when cyphers are placed on the right hand of numbers, they increase their value in a ten-fold proportion; but when placed to the right of decimals, they are of no signification. This will be sufficiently explained by a single example. Thus .0002 signifies only two parts of ten thousand; whereas without the cyphers it would have signified 2 tenths. On the contrary, the cyphers placed on the right hand of the whole numbers increase their value in a ten-fold proportion; for 2000, is two thousand; that is, the value of the figure is increased from two to two thousand by the addition of three cyphers: whereas .2000, signifies only two tenths, being of the same value either with or without the cyphers.

Disc. You are extremely kind in being so very explicit in your instructions. Is there any thing farther to be observed in the preceding table?

Philo. Yes, my dear Discipulus. You may observe, that all whole numbers are in effect nothing more than decimal parts to one another. For in any rank or series of equal numbers, as 666, the second 6 towards the left hand is ten times the value of the next 6 in the last place, or on the right hand of it, and but the tenth part of the 6 on the left. Either of them, therefore, may be taken as integers, or parts of an integer. If integers, they must be set down without a separat-

ing point between them, thus 666. But if they are partly integers, and partly decimals, a separating point must be placed between them, as 66.6, or 6.66. In the former the value is 66 and 6 tenths; in the latter, 6 and sixty-six hundred parts of an unit. But they are still in a ten-fold proportion to one another.

From these observations, compared with the preceding table, it will be easy to conceive, that decimal parts take their denomination from the place of the last figure: that is, .5 is $\frac{5}{10}$, .56, is $\frac{56}{100}$, .056, is $\frac{56}{1000}$. That .5 is five tenths, .05 is five parts of a hundred, .005, is five parts of a thousand, .0005, is five parts of ten thousand, &c.

Disc. I think you have made every particular extremely plain, and I will take care to imprint your instructions in my memory.

D I A L O G U E IV.

Reduction of Decimals.

Discipulus. **W**HAT do you mean by Reduction of Decimals?

Philo. Reduction is the rule, by which any vulgar fraction is reduced or changed into a decimal of equal value; or the value of a decimal, in the known parts of coin, weight, measure, &c. is discovered. I shall begin with the first, *viz.* the method of reducing a vulgar fraction into a decimal of the same value, and then proceed to explain the manner of finding the value of any decimal fraction.

I. To reduce a Vulgar Fraction into a Decimal.

This part of Reduction is performed by the following general

R U L E.

Add cyphers at pleasure to the numerator, and divide by the denominator; the quotient will be the decimal equivalent in value to the given vulgar fraction, observing to prick off as many figures in the quotient, as you added cyphers to the numerator.

Ex-

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EXAMPLES.

Reduce $\frac{1}{2}$ into a decimal.

$$2) 10 (5$$

In the same manner $\frac{1}{4}$ will be found equal to .25,
 $\frac{3}{4}$ equal to .75, and $\frac{5}{8}$ to .625,

Reduce $\frac{9}{56}$ into an equivalent decimal.

$$56) 9.00000000 (.16071428$$

56

340

336

..400

392

...80

56

240

224

..160

112

..480

448

..32

Reduce $\frac{6}{17}$ into an equivalent decimal.

$$17 \overline{) 6.0000} \quad (.3529$$

51

90

85

50

34

160

153

7

Disc. I see no difficulty in performing these operations; but may it not sometimes happen, that there are fewer figures in the quotient, than there are cyphers added to the dividend?

Philo. Yes, this is often the case: but whenever it happens, a cypher, or cyphers must be placed on the left hand of the first figure; there being a necessity that the figures, with cyphers if necessarily prefixed, should be equal in number, to the cyphers added to the numerator in the dividend. I will give you an example of this kind.

Let it be required to reduce $\frac{1}{16}$ to an equivalent decimal.

$$16 \overline{) 1.0000} \quad (.0625$$

96

40

32

80

80

00

You

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You observe that in the quotient there are only three figures, whereas four cyphers are added in the dividend; I therefore prefix a cypher before the six, and the decimal required is .0625.

Disc. I am greatly obliged to you, Sir, and think I now understand this rule: but I should be glad to know why there is often a remainder after the division is finished?

Philo. The remainder after the division shews, that the decimal is not precisely equal to the given vulgar fraction: but by continuing the division, it may be carried to what degree of accuracy you please, though, in many cases, it can never be exact.

Disc. I think I have heard those, who are skilled in this part of arithmetic, mention circulating decimals; will you please to tell me, what they mean by this term?

Philo. With all my heart, Discipulus. What is called circulating, or repeating decimals, are nothing more than a certain number of figures, which constantly return, as long as the division is continued. An example or two will make this extremely plain.

Let it be required, to reduce $\frac{1}{13}$ into an equivalent decimal.

$$13) 1.000000, \&c. (.07692307692307, \&c.$$

91

—
·90

78

—
120

117

—
·30

26

—
·40

39

—
·10, &c.

Here you observe, that the figures .076923, are repeated, and would continue to be again repeated if the division were pursued.

Let it be required to reduce $\frac{2}{3}$ into an equivalent decimal.

6) 2.000, &c. (.333, &c.

18 ..

20

18

20

18

20

In this instance the repetend or circulating decimal is 3, and may be continued to any degree of accuracy desired.

Disc. This is very satisfactory, Sir, and I shall always take care, as soon as I perceive that the figures, in the quotient, return in the same order, not to pursue the work any farther, as I plainly perceive they may be extended without the trouble of dividing.

Phil. You are extremely right, Discipulus; and it gives me pleasure to find you are so attentive to my instructions: I will add, that if you continue the same assiduity, you will soon become a complete measurer.

Disc. I should think myself undeserving of your favours, if I did not endeavour to turn them to my own profit. I have indeed often heard the master blamed for the small progress made by his scholars; but am persuaded, the fault is generally more owing to the inattention of the latter, than to the want of assiduity in the former.

Phil. I believe this is often the case.—But I shall now shew you how to reduce any known parts of coin, weights, measures, &c. into decimal parts. And to make this easy, you must observe the following

RULE.

RULE.

Reduce the given parts of coin, weight, measures, &c. into a vulgar fraction, whose denominator is the number of those known parts contained in the integer, and the given parts its numerator.

Let it, for instance, be required to reduce 16s. 6d. to the decimal of a pound sterling?

In order to solve this question, you must reduce both the numerator and denominator into six-pences, because that is the lowest denomination in the numerator.—Therefore 33 six-pences, which are equal to 16s. 6d. make the numerator; and 40, the number of six-pences in a pound sterling, the denominator; and then the question will stand thus $\frac{33}{40}$, and the decimal easily found as before.

$$\begin{array}{r}
 40 \overline{) 33.000} \quad (.825. \\
 \underline{320} \quad \cdot \cdot \\
 \cdot 100 \\
 \underline{80} \\
 \cdot 200 \\
 \underline{200} \\
 \cdot \cdot \cdot
 \end{array}$$

Therefore .825, is the decimal required.

EXAMPLE II.

Let it be required to reduce 13s. 4d. into the decimal of a pound sterling.

Here the 13s. 4d. must be reduced into pence, and will make 160: but the pence in a pound sterling are 240; and therefore the vulgar fraction will be $\frac{160}{240}$, or $\frac{2}{3}$, and the decimal .6666, &c. as you will easily find by dividing the numerator by the denominator.

Ex-

EXAMPLE. III.

Let it be required to reduce 3 inches to the decimal of a foot.

Here the vulgar fraction is evidently $\frac{3}{12}$, because twelve inches make a foot: and 3, with cyphers annexed, divided by 12, gives .25, the decimal required.

EXAMPLE IV.

Reduce 7 inches $\frac{3}{4}$ to the decimal of a foot.

Here the numerator will be 31, there being 31 quarters in $7\frac{3}{4}$ inches; and the numerator 48; because 48 quarters of an inch make a foot. Then 31 divided by 48 will give .64583, &c. the decimal required.

$$\begin{array}{r} 48 \overline{) 31.00000} \quad (.64583, \&c. \\ \underline{288} \quad \dots \end{array}$$

• 220

192

• 280

240

• 400

384

• 160

144

16, &c.

I hope, my dear Discipulus, you have attentively considered these examples, which I think sufficient for the purpose: but as an exercise for you when alone, I shall add the following examples with their answers, leaving the arithmetical operations to yourself.

Decimals of a Foot.

$\frac{1}{4}$ Inch	— is —	.02083.
$\frac{1}{2}$ —	— —	.0416.
$\frac{3}{4}$ —	— —	.0625.
1 Inch	— —	.083.
2 Inches	— —	.16.
3 —	— —	.25.
4 —	— —	.333 &c.
5 —	— —	.416
6 —	— —	.5.
7 —	— —	.583.
8 —	— —	.666.
9 —	— —	.75.
10 —	— —	.83.
11 —	— —	.916.

Decimals of a Pound Sterling.

1 Farthing	— is —	.001 &c.
2 Farthings	— —	.002.
3 —	— —	.003.

A L S O,

10s. 8d.	— is —	.533 &c.
13 : 10 $\frac{1}{2}$	— —	.69375.
15 : 9 $\frac{3}{4}$	— —	.790625.
19 : 11 $\frac{1}{4}$	— —	.996875.
1 : 10 $\frac{1}{4}$	— —	.0927083.

These examples being well considered, and the operations at large often performed, will render it extremely easy for you to reduce any known parts of an integer into decimal parts.

Diff. I will carefully study these examples, and do not doubt of being soon master of the whole. But I would ask one question: it is this, whether the numerator and denominator **must not always** be reduced into the lowest denomination of the former? For instance, must not the 15s. 9d. $\frac{3}{4}$. in one of the above examples, be reduced into farthings? If so, the vulgar fraction will stand thus $\frac{759}{800}$, because 759 farthings make 15s. 9 $\frac{3}{4}$ d. and 960 farthings make a pound.

Philo. You are extremely right; and it gives me the highest pleasure to find you so readily comprehend my meaning. I shall therefore now proceed to the second part of Reduction of Decimal Fractions, or that of finding their value.

II. *To find the Value of a Decimal in the known Parts of Money, Weights, Measures, &c.*

This part of Reduction is nothing more than the converse of the former, and is performed by the following

R U L E.

Multiply the decimal by the next inferior denomination, and prick off, from the product as many figures or cyphers, as there were figures, &c. in the given decimal: multiply the figures so pricked off by the number of figures in the next inferior denomination, and prick off the figures, &c. as before; proceeding in this manner, till you arrive at the least denomination. Then are the figures cut off on the left hand equivalent to the given decimal.

E X A M P L E S.

What is the value of 0.825 decimals of a pound sterling?

O P E R A T I O N.

$$\begin{array}{r}
 \text{c.825} \\
 \text{20 Shillings in a Pound,} \\
 \hline
 \text{Shillings} \quad 16.500 \\
 \text{12 Pence in a Shilling.} \\
 \hline
 \text{Pence} \quad 6.000 \\
 \text{Answer 16s. 6d.}
 \end{array}$$

Here you observe, that I multiply the given decimal 0.825, by 20, because 20 shillings make a pound, and prick off three decimals in the product; and the answer is 16 shillings, and .500 decimal parts, which being multiplied by 12, the number of pence in a shilling,

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and three decimals pricked off, the answer will be sixpence. Had the decimals remaining been figures instead of cyphers, they must have been multiplied by 4, the number of farthings in a penny; but as the cyphers are of no value, the question is finished, and 16s. 6d. the true value of the given decimal.

Let it be required to find the value of 0.7471, decimal parts of a pound.

OPERATION.

$$\begin{array}{r}
 0.7471 \\
 \underline{20} \\
 14.9420 \\
 \underline{12} \\
 11.3040 \\
 \underline{4} \\
 1.2160
 \end{array}$$

Answer 14s. 11 $\frac{1}{4}$ d. nearly.

What is the value of 0.74722 decimal parts of a pound, troy weight?

OPERATION.

$$\begin{array}{r}
 0.74722 \\
 \underline{12 \text{ Ounces in a pound Troy.}} \\
 8.96664 \\
 \underline{20 \text{ Penny-weights in an ounce.}} \\
 19.33280 \\
 \underline{24 \text{ Grains in a penny-weight.}} \\
 133120 \\
 \underline{66560} \\
 7.98720
 \end{array}$$

Answer 8 ounces, 19 penny-wts. 8 grains nearly.

Disc. Why do you say 8 grains nearly, when there is only 7 pricked off, in the place of grains?

Philo. It gives me pleasure you have taken notice of it: I intended to have explained the reason why I made

made it 8 instead of 7 grains in the answer. In order to this you must observe, that though only 7 is separated, there still remains a large decimal, amounting to more than 9 tenths of another grain: consequently 8 grains is much nearer the truth than 7.

Disc. I thank you, Sir, and find that the real value of a decimal cannot be found, till the decimal, after multiplication, becomes a cypher or cyphers.

Philo. Extremely right: and you will observe, that in the question preceding the last, the value of the given decimal is not accurately found: but as the decimal after the last operation is little more than two tenths, or one fifth of a farthing, it is of too little consequence to be regarded.

Disc. I shall very carefully attend to your instructions, and should be obliged to you for a few more examples, that I may, by practice, become perfect in this part of decimal arithmetic, as I perceive already, it is of the utmost consequence to be well understood.

Philo. I will very readily give you other examples; and in order to render them of more advantage, shall only give the answers.

What is the value of 0.0095 decimal parts of a pound sterling?

Answer, $2 \frac{1}{4} d.$

What is the value of 0.0428 of a pound sterling?

Answer, $10 \frac{1}{4} d.$

What is the value of 0.87708 of a pound sterling?

Answer, 17s. $6 \frac{1}{2} d.$

What is the value of 0.87628 of a pound Troy?

Answer, 10 oz. 10 pwts. 7 grs.

What is the value of 0.798645 of a hundred weight avoirdupoise?

Answer, 3 qrs. 5 lb. 7 oz.

What is the value of 0.9595 of a foot?

Answer, 11 inches and 2 qrs.

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DIALOGUE V. SECT. I.

Addition of Decimals.

Philo. **I** Hope, my dear Discipulus, you are now master of the questions I gave you at our last meeting, and therefore shall now proceed to Addition of Decimals, which is performed by the following

RULE.

Set down the numbers proposed to be added, in such a manner, that those of the same value may stand directly under each other, whether they be mixed numbers, or decimal parts; which may be very easily done, if you place the separating points in every line under one another. Then, add them as if they were all whole numbers, separating from the sum as many figures, &c. as there are decimals in the greatest of the given expressions.

EXAMPLES.

$$\begin{array}{r} 34.5 \\ 65.3 \\ 128.74 \\ 95.0 \\ 87.8 \\ 7.9 \end{array}$$

419.24 the sum required.

$$\begin{array}{r} 45.07 \\ 50.758 \\ 123.0057 \\ 75.702 \\ 24.8 \end{array}$$

319.3357 Answer.

$$\begin{array}{r} 574.678953 \\ 95.79643 \\ 78.0546 \\ 54.789 \\ 8.9 \end{array}$$

812.218983 Answer.

1.957642.

1.975642

-745257

.000598

.8007

.64053

 4.162727 Answer.

Disc. If all the rules of decimal arithmetic, be attended with no more difficulty than this of Addition, I hope soon to be master of that useful method of computation: for I observe, that the only caution necessary, is to place separating points directly under one another; the whole sum being added, as in common arithmetic, without any regard to the separating points, till the operation is finished, and then to separate as many figures towards the right hand, as there are in the greatest number. Thus in the first example, because there are two decimals in the third line, two figures are separated towards the right hand in the total.

Philo. You are extremely right, Discipulus: and, as there are no exceptions to this general rule, we will proceed to the next in order.

S E C T. II.

S U B T R A C T I O N.

This, like the former, is performed, in all respects, like Subtraction in whole numbers; and the rule respecting the points, and the number of figures to be separated in the remainder, are the same as before in Addition.

E X A M P L E S.

From 74.918765
Take 69.180714

 Remains 5.738051

From 8.100007
Take 6.9718743

 Remains 1.1281327

From

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$$\begin{array}{r} \text{From } 28.007 \\ \text{Take } 6.98798012 \\ \hline \text{Remains } 21.01901988 \end{array}$$

$$\begin{array}{r} \text{From } 77.38160007 \\ \text{Take } 4.9812 \\ \hline \text{Remains } 72.40040007 \end{array}$$

Disc. This is indeed very plain; and I will endeavour to imprint it in my mind by frequent practice.

Philos. A very proper resolution, Discipulus; for without practice it is impossible to become an expert arithmetician. Rules without practice are soon forgotten, and errors are perpetually committed by those who are not accustomed to operations.

S E C T. III.

MULTIPLICATION.

Multiplication of decimals is also performed in the same manner, as it is in whole numbers; but in order to find the true value of the product, you must observe the following

R U L E.

Separate as many figures towards the right hand in the product, as there are decimal parts in both the multiplicand and multiplier added together. But if there are not so many places in the product, the defect must be supplied by cyphers on the left hand. A few examples will sufficiently explain this rule.

EXAMPLE I.

$$\begin{array}{r} 123.024 \\ 2.232 \\ \hline 246048 \\ 369072 \\ 246048 \\ 246048 \\ \hline 274.589568 \end{array}$$

Here

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Here, because there are three decimals in the multiplicand, and three in the multiplier, I separate six figures towards the right hand in the product.

EXAMPLE II.

$$\begin{array}{r}
 3684.7928 \\
 84.216 \\
 \hline
 221087568 \\
 36847928 \\
 73695856 \\
 147391712 \\
 294783424 \\
 \hline
 310318.5104448
 \end{array}$$

EXAMPLE III.

$$\begin{array}{r}
 .2365 \\
 .2435 \\
 \hline
 11825 \\
 7095 \\
 9460 \\
 4730 \\
 \hline
 .05758775
 \end{array}$$

In the last example you will observe, that there are four places of decimals in the multiplicand, and the same number in the multiplier, therefore eight places must be separated in the product; but as there are but seven, a cypher is added on the left hand to make up the number.

Disc. I hope I thoroughly understand your meaning; but should be glad to know the reason for placing the cypher on the left hand of the product.

Phil. The reason is extremely plain: any number multiplied by a fraction, or what is less than a unit, will be less after the operation than before. Thus, if 25 be multiplied by .5 or one half; the product will be only 12.5, or 12 and a half, and if 25 were to be multiplied by .05, the product would be no more than 1.25, or one and a quarter. But in the above example both the multiplier and multiplicand are less than unity; and

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and consequently they must produce a number much less than unity. This will plainly appear from the following

EXAMPLE IV.

$$\begin{array}{r}
 .0347 \\
 .0236 \\
 \hline
 2082 \\
 1041 \\
 694 \\
 \hline
 .00081392
 \end{array}$$

Here the places in the product are only five, whereas there are eight decimals in the multiplicand and multiplier; three cyphers are therefore added to supply the defect.

I shall add the following examples as an exercise to these rules, and hope you will carefully perform the operations.

EXAMPLES.

1. Multiply 57.056, by 0.778. Answer 44.389568.
2. Multiply 7.6543, by 5.4246. Ans. 41.52151578.
3. Multiply 0.56879, by 0.05674.
Answer 0.0322731446.
4. Multiply 0.03246, by 0.02364.
Answer 0.0007673544.
5. Multiply 87649, by 0.03687. Ans. 3231.61863.
6. Multiply 94.35786, by 6.57869.
Answer 620.7511100034.
7. Multiply 3.141592, by 52.7438.
Answer 165.6995001296.

Disc. You are very kind, Sir, and as you have already explained the manner of pricking off the decimals, I shall be very soon able to work the above questions at large. I observe, that in the fifth example, the multiplicand is entirely composed of whole numbers, and the multiplier wholly of decimals. The operation I perceive is however the same, because you have

have separated five places on the right hand, the number of decimals contained in the multiplier.

Philo. You have made a very just observation, Discipulus. It gives me great pleasure to find that you retain the rules so firmly in your memory. They are indeed very easy, and therefore I have not so often repeated them, as I may perhaps think necessary hereafter. But you will at present observe, that in multiplying decimals by decimals, or where there are several places of decimals, there are often many more decimals than are necessary in the product. For in Practice three or four places of decimals are generally sufficient; and a greater number is troublesome and unnecessary. I shall therefore shew you the proper method of proceeding in cases of this nature; by which means great part of the labour will be saved, and yet the question be answered sufficiently accurate. This useful contraction is performed by the following

R U L E.

Set the unit's place of the multiplier directly under that figure in the multiplicand, whose place you intend to keep in the product; and invert the order of all its places; that is, write the decimals on the left, and the integers, if any, on the right.

Then, in multiplying, omit all the figures placed, in the multiplicand, to the right of the digit, with which you begin the operation, and let the right hand place of every line stand under each other.

But be sure to remember, that the first figure in each line of the operation be increased by the carriage, which would arise from the places omitted; that is, carrying 1 from 5 to 15; 2 from 15 to 25; 3 from 25 to 35, &c. instead of carrying one for every ten, &c. and the sum of these lines will give the product generally exact.

A few examples will make this rule very easy, if you take care to imprint it in your memory.

E X A M P L E I.

Let it be required to multiply 3.141592 by 52.7438; and let there be only four decimal parts in the product.
These

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These numbers, according to the above rule, must be placed thus :

3.141592	The multiplicand, in its natural order.
<u>8347.25</u>	The multiplier inverted.
1570796	The prod. by 5, with carriage of 5 times 2.
62832	The prod. by 2, with carriage of 9 times 2.
21991	Product by 7, with its proper increase.
1257	Prod. by 4, &c.
94	Prod. by 3, &c.
25	Prod. by 8, &c.
<u>165.6995</u>	The true product required.

The reason of this contraction will be very obvious from the whole operation being wrought at large, in the following manner.

$$\begin{array}{r}
 3.141592 \\
 \underline{52.7438} \\
 25 \overline{) 132736} \\
 94 \overline{) 24776} \\
 1256 \overline{) 6368} \\
 21991 \overline{) 144} \\
 62831 \overline{) 84} \\
 1570796 \overline{) 0} \\
 \hline
 165.6995 \overline{) 001296}
 \end{array}$$

From this example it is evident, that all the figures on the right hand of the line through the operation are omitted in the former contracted method ; and that the last single product here, is the first there ; though the result is equal in both as far as four decimal places.

You will also observe, that because four decimal places only are to be kept, the multiplier 2, is placed under 5, the fourth place in decimals in the multiplicand.

EXAMPLE II.

Multiply 384.672158 by 36.83645 ; and let only four Decimal Places be kept in the Product.

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$$\begin{array}{r}
 384.672158 \text{ Multiplicand.} \\
 54638.63 \text{ Multiplier inverted.} \\
 \hline
 115301647 \\
 23080329 \\
 3077377 \\
 115402 \\
 15387 \\
 1924 \\
 \hline
 14169.2066 \text{ Product required.}
 \end{array}$$

Had this question been solved in the common manner, it would have stood thus :

$$\begin{array}{r}
 384.672158 \\
 36.8345 \\
 \hline
 1923 \mid 360790 \\
 15386 \mid 88632 \\
 115401 \mid 6174 \\
 3077377 \mid 264 \\
 23080329 \mid 48 \\
 115401617 \mid 4 \\
 \hline
 14169.2066 \mid 038510
 \end{array}$$

Discipulus. I think, Sir, I understand your meaning ; but if you will please to work over one or two of the first figures in the multiplier, it will perhaps render it still plainer.

Philo. With all my heart, *Discipulus.* I shall be always ready to remove every difficulty that may in the least retard your progress.

We will begin with the first line ; where I say, 3 times 8 is 24, and carry 2 ; 3 times 5 is 15 and 2 is 17, now set down 7 and carry 1, &c. because this is the product arising from the multiplication of the 5 that stands over the 3, the other being omitted as of no value, but to know what is to be carried : the remaining part of the line is the same as in common multiplication.

In the second line I say 6 times 8 is 48, and carry 5, because greater than 45, otherwise only 4 must have been carried ; 6 times 5 is 30, and 5 is 35, carry 3 ; 6 times 1 is 6, and 3 is 9 : Now being come to the figure over the 6, set down 9.

In

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In the third line I say, 8 times 5 is 40, and carry 4; 8 times 1 is 8, and 4 is 12, and carry 1; 8 times 2 is 16, and 1 is 17: now being come to the figure over 8, set down 7, and carry 1, &c. In this manner I proceed with every figure in the inverted multiplier, till the whole work is finished.

Discipulus. I am extremely obliged to you, Sir, and shall be able to pursue your instructions without any farther difficulty.

Philo. I hope you will, Discipulus; but in order to assist you still farther, I will add a few examples, and where I think the least difficulty occurs, I will add the work at large.

EXAMPLE III.

Let it be required to multiply 395.3758, by .75642, and reserve only four decimal places in the product.

Contracted Operation.

$$\begin{array}{r}
 395.3758 \\
 \cdot 24657 \\
 \hline
 2767629 \\
 197683 \\
 23722 \\
 1581 \\
 76 \\
 \hline
 299.0696
 \end{array}$$

EXAMPLE IV.

Multiply 257.386 by 76.48, and reserve only the entire product of Integers.

Contracted Operation.

$$\begin{array}{r}
 257.386 \\
 84.67 \\
 \hline
 18015 \\
 1544 \\
 103 \\
 20 \\
 \hline
 19682
 \end{array}$$

C 2

Operation

Operation at large.

$$\begin{array}{r}
 257.386 \\
 76.48 \\
 \hline
 20 \overline{) 58848} \\
 102 \overline{) 9424} \\
 1544 \overline{) 136} \\
 18014 \overline{) 92} \\
 \hline
 19682 \overline{) .58688}
 \end{array}$$

S E C T. IV.

DIVISION of DECIMALS.

Philo. Having, my dear Discipulus, conducted you through the three first rules of decimals, I shall now proceed to Division, the fourth rule, and indeed the most difficult of the whole; but it will soon be learned, if attention be not wanting.

Discipulus. I will promise to listen attentively to your instructions, and as you have been so indulgent as to remove even the appearance of difficulty, I make no doubt of being soon able to comprehend this rule, as well as the others.

Philo. I doubt it not, Discipulus; nor shall the plainest instructions in my power be wanting for this purpose; and in order to this, I shall first give a general rule for discovering the true value of the quotient figures.

GENERAL RULE.

The places of decimal parts in the divisor and quotient, being counted together, must be always equal in number to those in the dividend.

This rule admits of four cases.

C A S E I.

When the places of decimals in the divisor and dividend are equal, the quotient will be whole numbers.

EXAMPLES.

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EXAMPLES.

Divide 295.75 by 8.45.

8.45) 295.75 (35 The answer.

$$\begin{array}{r} 2535 \\ \underline{4225} \\ 4225 \end{array}$$

....

Divide .4368 by .0078.

.0078) .4368 (56 The answer.

$$\begin{array}{r} 390 \\ \underline{.468} \\ .468 \\ \underline{.468} \\ .000 \end{array}$$

CASE II.

When the places of decimal parts in the dividend exceed those in the divisor, cut off the excess for decimal parts in the quotient.

EXAMPLES.

24.3) 786.516 (32.12:

$$\begin{array}{r} 729 \dots \\ \underline{.515} \\ 486 \\ \underline{.291} \\ 243 \\ \underline{486} \\ 486 \\ \underline{.000} \end{array}$$

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$$436) 34246.056 \text{ (} 78.546 \text{.)}$$

$$\underline{3052} \dots$$

$$\cdot 3726$$

$$\underline{3488}$$

$$\cdot 2380$$

$$\underline{2180}$$

$$\cdot 2005$$

$$\underline{1744}$$

$$\cdot 2616$$

$$\underline{2616}$$

$$\dots$$

$$\cdot 534) \cdot 30438 \text{ (} \cdot 57 \text{.)}$$

$$\underline{\cdot 2670}$$

$$\cdot 3738$$

$$\underline{3738}$$

$$\dots$$

CASE III.

When there are not so many places of decimals in the dividend as there are in the divisor, annex cyphers to the dividend, to render them equal, and the quotient will be whole numbers.

EXAMPLES.

Divide 192.1 by 7.684. And 441 by .7875.

7.684) 192.100 (25 the quotient required.

$$\underline{15368}$$

$$\cdot 38420$$

$$\underline{38420}$$

$$\dots$$

$$\cdot 7875) 441.0000 \text{ (} 560 \text{.)}$$

$$\underline{39375}$$

$$\cdot 47250$$

$$\underline{47250}$$

$$\dots$$

CASE IV.

If after the division is finished, there are not so many figures in the quotient as there ought to be places of decimals by the general rule, supply their defect by prefixing cyphers to it.

EXAMPLE I.

Divide 7.25406, by 957

957) 7.25406 (.00758.

$$\begin{array}{r}
 6699 \quad \cdot \cdot \\
 \hline
 \cdot 5550 \\
 4785 \\
 \hline
 \cdot 7656 \\
 \cdot 7656 \\
 \hline
 \cdot \cdot \cdot \cdot
 \end{array}$$

Here you will observe, that the quotient is 758; but as there are five places of decimals in the dividend, and none in the divisor, five places must be pricked off in the quotient. I therefore prefix two cyphers to the figures in the quotient, and it becomes .00758, the true answer.

EXAMPLE II.

Divide .7183241, by 7.2.

7.2) .71832410 (.0997672.

$$\begin{array}{r}
 648 \quad \cdot \cdot \cdot \cdot \cdot \\
 \hline
 \cdot 703 \\
 648 \\
 \hline
 \cdot 552 \\
 504 \\
 \hline
 \cdot 484 \\
 432 \\
 \hline
 \cdot 521 \\
 504 \\
 \hline
 \cdot 170 \\
 144 \\
 \hline
 \cdot 26 \\
 \hline
 C. 4
 \end{array}$$

Discipulus.

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Discipulus. You have made this rule very easy by dividing it into four Cases, which would otherwise, I perceive, have been very difficult.

Philo. It gives me pleasure to observe, that the method I have taken, has rendered this rule easy to be attained; especially as it is the most difficult part of Decimal Arithmetic. I shall add a few examples, containing all the varieties that can possibly happen in division of decimals, leaving the operations at large as an exercise.

Let it be required to divide 49306.6, by 57.4.

Answer, 859.

Divide 493.066, by 574. Answer, .859.

Divide 4930.66, by 5.74. Answer, 859.

Divide 49.3066, by 5.74. Answer, 8.59.

Divide 493.066, by .574. Answer, 85930.

Divide 493.066, by .0574. Answer, 8590.

Divide 493066, by .0574. Answer, 8.59.

Discipulus. I shall very soon be able to work these examples at large; for I perceive, that the figures in both dividend, divisor, and quotient, are the same in all; and differ only in placing the points.

Philo. They are all the same, and intended only to shew the different values of the same figures, when the points are put in different places; and which I hope you will carefully observe.

In treating of Multiplication, I shewed you a method of contracting the work, by using no more figures than were necessary for finding the true value of the product, to any given number of decimal places. I will now shew you how you may contract the work in Division: and in order to this, it will be necessary to observe the following

R U L E.

Let each remainder be a new dividend, and for each of these new dividends, point off one figure from the right hand of the divisor; observing at each multiplication, to have regard to the increase of the figures so cut off, as before in contracted multiplication.

EXAMPLE

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EXAMPLE I.

The Work contracted.

$$\begin{array}{r}
 7.9863) 70.2300 (8.7938 \\
 \underline{638904} \\
 \cdot 63396 \\
 \underline{55904} \\
 \cdot 7492 \\
 \underline{7187} \\
 \cdot 305 \\
 \underline{239} \\
 \cdot 66 \\
 \underline{64} \\
 \underline{2}
 \end{array}$$

The work at large.

$$\begin{array}{r}
 7.9863) 70.23000000 (8.7938 \\
 \underline{638904} \cdot \cdot \cdot \cdot \\
 \cdot 633960 \\
 \underline{559041} \\
 \cdot 749190 \\
 \underline{718767} \\
 \cdot 304230 \\
 \underline{239589} \\
 \cdot 646410 \\
 \underline{638904} \\
 \underline{07506}
 \end{array}$$

I shall give two more examples, but without adding the work at large, thinking it entirely unnecessary.

INTRODUCTION.

EXAMPLE II.

9.365407) 87.076326 (9.297655.

$$\begin{array}{r}
 \cdot \cdot \cdot \cdot \cdot \quad 84288663 \\
 \cdot 2787663 \\
 \hline
 1873081 \\
 \hline
 \cdot 914582 \\
 842886 \\
 \hline
 \cdot 71696 \\
 65558 \\
 \hline
 \cdot 6138 \\
 5619 \\
 \hline
 \cdot 519 \\
 468 \\
 \hline
 \cdot 51 \\
 47 \\
 \hline
 4
 \end{array}$$

2.756756) 7414.76717 (2689.67118.

$$\begin{array}{r}
 \cdot \cdot \cdot \cdot \cdot \quad 5513512 \cdot \cdot \\
 19012551 \\
 16540536 \\
 \hline
 \cdot 24720157 \\
 22054048 \\
 \hline
 \cdot 2666109 \\
 2481080 \\
 \hline
 \cdot 185029 \\
 165405 \\
 \hline
 \cdot 19624 \\
 19277 \\
 \hline
 \cdot \cdot 327 \\
 276 \\
 \hline
 \cdot 51 \\
 28 \\
 \hline
 23 \\
 22 \\
 \hline
 \cdot 1
 \end{array}$$

Discipulus.

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Discipulus. I observe, Sir, that in the last example, the whole work is not contracted, but a part only; will you be kind enough to tell me the reason for that particular.

Philo. With all my heart: I made use of an example of that kind, that you might observe the difference. When there are more figures than the divisor multiplied by the first quotient figure will extend to, they must be brought down, and the work continued in the common manner; but after this, the work is performed in the same manner as in other examples.

Discipulus. You have entirely removed this difficulty, Sir: But I should be glad to know the reason for pricking off the decimals in all these contracted examples; I cannot reconcile it with the general rule.

Philo. A very little consideration, and working the operations at large, would have sufficiently removed this difficulty, which will immediately vanish when it is considered, that though there is no figure actually brought down; yet a cypher is supposed to be added to each new dividend. When these are counted with the other decimals in the dividend, you will find the decimals in the quotient to agree exactly with the general rule.

Discipulus. I thank you, Sir; and now see evidently the reasons on which all the operations are founded.

DIALOGUE V. SECT. I.

Extraction of the SQUARE ROOT.

Philo. **H**AVING explained every thing necessary in decimal arithmetic, I shall now proceed to the extraction of the square root, a rule necessary to be known by all who are desirous of knowing the reasons on which many of the operations in practical measuring are founded.

Discipulus. Pray what do you mean by extracting the Square Root?

Philo. By extracting the Square Root, is meant, the finding such a number, which being multiplied by itself, the Product shall be equal to the given number. Thus suppose 16 the given number, then will the square root

of that number be 4; because 4 multiplied by itself will produce 16. In this operation the given number is generally called the *resolvend*; and the root is extracted by the following general

R U L E.

1. Make a point over the place of units, and also over every second place, counting from unit's, to the left hand, for integers, and to the right hand for decimals; and the root will always consist of as many figures, as there are points over the *resolvend*. Each figure thus pointed, and the next to it towards the left hand, is called a period.

2. Seek the greatest square number in the left hand period, write the root in the quotient, and place the square thereof under the period: subtract the square out of the period, and to the remainder bring down the next period as in division, which will form a dividend.

3. Double the root, and place it to the left of the dividend, for a divisor; seek how often the divisor is contained in the dividend, omitting its right hand place; write the result in the quotient, and also on the right of the divisor.

4. Multiply this increased divisor by the last quotient figure: subtract the product from the dividend, and to the remainder bring down the next period for a new dividend; double the last figure in the last divisor, which with the other figure, or figures, will form a new divisor: divide with the same caution as before, and proceed in this manner till all the periods are finished.

A few examples will make this rule very easy.

EXAMPLE I.

Let it be required to extract the square root out of 572199960721.

This *resolvend* being pointed as directed in the rule, will stand thus:

572199960721

572199960721 (756439
49, the greatest square in 57.

$$\begin{array}{r}
 1 \text{ Divisor } 145 \overline{) 821} \\
 \underline{5} \\
 2 \text{ Div. } 1506 \overline{) 725} \\
 \underline{6} \\
 3 \text{ Div. } 15124 \overline{) 9699} \\
 \underline{4} \\
 4 \text{ Div. } 151283 \overline{) 9036} \\
 \underline{3} \\
 1512869 \overline{) 66396} \\
 \underline{13615821} \\
 13615821
 \end{array}$$

Thus is the whole operation finished, and the root required is 756439.

Discipulus. You have rendered the rule much plainer than I thought possible to be done by one example: but, as there are a great many figures, and the operation somewhat tedious, it is very possible that some mistakes may be committed: Is there no way to prove the truth of the operation?

Philo. Yes, my dear Discipulus, there is a very easy method of proving all operations of this kind. It consists in multiplying the root by itself, and adding the remainder, if any, and the product will give the resolvend. Thus if you multiply by itself 756439, the product, will be 572199960721, the given resolvend.

EXAMPLE II.

What is the square root of 1850701.764025?

OPERATION.

OPERATION.

$$\begin{array}{r}
 \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \\
 1850701.764025 \text{ (1360.405.} \\
 \hline
 1 \\
 23 \overline{) \cdot 85} \\
 \underline{3 } 69 \\
 266 \overline{) 1670} \\
 \underline{6 } 1596 \\
 27204 \overline{) \cdot \cdot 1101.76} \\
 \underline{4 } 108816 \\
 2720805 \overline{) \cdot \cdot 13604025} \\
 \underline{} 13604025 \\
 \cdot \cdot \cdot \cdot \cdot
 \end{array}$$

EXAMPLE III.

What is the square root of 0.06076225 decimal parts.

OPERATION.

$$\begin{array}{r}
 \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \\
 0.06076225 \text{ (0.2465. The root required.} \\
 \hline
 .04 \\
 44 \overline{) 207} \\
 \underline{4 } 176 \\
 486 \overline{) \cdot 3162} \\
 \underline{6 } 2916 \\
 4925 \overline{) \cdot 24625} \\
 \underline{} 24625 \\
 \cdot \cdot \cdot \cdot \cdot
 \end{array}$$

You will observe, my dear Discipulus, that in all the above examples, the given resolvends have been perfect squares; and therefore their roots have been extracted without any remainder: but it very often happens, that the resolvend is not a true square, and when that is the case, something will remain after the extraction is finished. Nor can the roots of these numbers be ever accurately found, though you may approach infinitely near

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near the truth, by annexing as many periods of cyphers as you please to the resolvend, and continuing the operation as before.

EXAMPLE IV.

Let it be required to extract the square root of 6968.

OPERATION.

$$\begin{array}{r}
 \dots \\
 6968 \text{ (83.4745, \&c.)} \\
 \underline{64} \\
 163 \overline{) 568} \\
 \underline{3 489} \\
 1664 \overline{) 79.00} \\
 \underline{4 6656} \\
 16687 \overline{) 124400} \\
 \underline{7 116809} \\
 166944 \overline{) 759100} \\
 \underline{4 667776} \\
 1669485 \overline{) 9132400} \\
 \underline{8347425} \\
 784275, \&c.
 \end{array}$$

And in this manner may the operation be continued as far as you please ; but the true root can never be accurately found.

EXAMPLE V.

Extract the square root out of 763958207163:

OPERATION.

INTRODUCTION.

OPERATION.

763958207163 (874047.027

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$$\begin{array}{r}
 167 \overline{) 1239} \\
 \underline{7 } \\
 1744 \overline{) 7058} \\
 \underline{4 } \\
 174804 \overline{) 822071} \\
 \underline{4 } \\
 1748087 \overline{) 12285563} \\
 \underline{7 } \\
 174809402 \overline{) 12236609} \\
 \underline{2 } \\
 1748094047 \overline{) 489540000} \\
 \underline{349618804} \\
 13992119600 \\
 12236658329 \\
 \hline
 1755461271, \&c.
 \end{array}$$

These examples, which are all worked at large, will, I hope, be sufficient to explain the method of extracting the square root out of any number; and therefore I shall only add the following examples, leaving the operations as an exercise for you.

EXAMPLES.

1. What is the square root of 36372961?
Answer 6031.
2. What is the square root of 24681024?
Answer 4968.
3. What is the square root of 1.0609?
Answer 1.03.
4. What is the square root of 911236798.794365?
Answer 30186.699.
5. What is the square root of 2990667969?
Answer 54687.
6. What is the square root of 7?
Answer 2.6457131106.
7. What is the square root of .125?
Ans. .3535.
8. What is the square root of .00715?
Ans. .084.

I hope

I N T R O D U C T I O N. xli

I hope you will be very attentive in working the above examples ; because it will render you a complete master of this useful rule. I shall now proceed to teach you the manner of extracting the cube root.

S E C T. II.

E X T R A C T I O N *of the* C U B E R O O T.

Discipulus. What do you mean by extraction of the cube root ?

Philo. By extraction of the cube root, is meant the finding such a number, which being multiplied by itself, and then multiplied by the above product, shall produce the given number. This is performed by the following

R U L E.

1. Make a point over the place of units, and also over every third place, counting to the left for integers, and to the right for decimals ; then will there be as many figures in the root, as there are points over the given number.

2. Seek the greatest cube in the first period, or that nearest the left hand ; write the root in the quotient, and the cube under the period ; subtract the cube from the period ; and to the remainder bring down the next period ; call it the resolvend, and draw a line under it.

3. Under this resolvend write the triple square of the root, so that units in the latter may stand under the place of hundreds in the former ; under this triple square write the triple root removed one place towards the right ; and the sum of these two numbers call a divisor, drawing a line under it.

4. Seek how often this divisor may be had in the resolvend, excepting its right hand place, and write the answer in the quotient.

5. Under the divisor write the product of the triple square of the root, by the last quotient figure, setting the units place of this line, under that of tens in the divisor : under this line write the product of the triple root, by the square of the quotient figure, placing this one place beyond the right of the former ; and under this line removed one place farther to the right, set down
the

the cube of the last quotient figure: add these three lines together, and call their sum the subtrahend, drawing a line under it.

6. Subtract the subtrahend from the resolvend; to the remainder bring down the next period for a new resolvend; the divisor to this must be the triple square of all the quotient added to the triple thereof, and proceed in every other respect as before.

Discipulus. This, Sir, is a very long rule, but I will endeavour to learn it with the greatest expedition.

Philo. A little attention will be sufficient for the purpose; especially after I have fully explained it, by a few examples.

EXAMPLE I.

Let it be required to extract the cube root out of the number 48228544 ?

$$\begin{array}{r}
 \begin{array}{r}
 \cdot \cdot \cdot \\
 48228544 \quad (364 \\
 27 \text{ The greatest cube in } 48. \\
 \hline
 21228 \text{ Resolvend.}
 \end{array} \\
 \text{Add } \left\{ \begin{array}{l}
 27 \text{ Triple square of the root } 3. \\
 9 \text{ Triple of } 3. \\
 \hline
 279 \text{ Divisor.}
 \end{array} \right. \\
 \text{Add } \left\{ \begin{array}{l}
 162 \text{ Triple square of } 3, \text{ mult. by } 6. \\
 324 \text{ Triple of } 3, \text{ mult. by square of } 6. \\
 216 \text{ Cube of } 6. \\
 \hline
 19656 \text{ Subtrahend.} \\
 1572544 \text{ Resolvend.}
 \end{array} \right. \\
 \text{Add } \left\{ \begin{array}{l}
 3888 \text{ Triple square of the root } 36. \\
 108 \text{ Triple of } 36. \\
 \hline
 38988 \text{ Divisor.}
 \end{array} \right. \\
 \text{Add } \left\{ \begin{array}{l}
 15552 \text{ Triple square of } 36, \text{ mult. by } 4. \\
 1728 \text{ Triple of } 36, \text{ mult. by } 4. \\
 64 \text{ Cube of } 4. \\
 \hline
 1572544 \text{ Subtrahend.}
 \end{array} \right.
 \end{array}$$

In the above example, the given number is a true cube; for when the last subtrahend is subtracted from the

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the last resolvend, there will be no remainder : and the work may be proved by multiplying the root into itself, and that product again by the root, when the last product will be the same with the given number. See the work.

$$\begin{array}{r}
 364 \\
 \underline{364} \\
 1456 \\
 2184 \\
 \underline{1092} \\
 132496 \\
 \underline{364} \\
 529984 \\
 794976 \\
 \underline{397488} \\
 48228544 \text{ Proof.}
 \end{array}$$

Discipulus. I am greatly obliged to you, Sir, for performing this operation in so plain and instructive a manner: It has explained the rule you at first laid down in a very clear manner: and if you will indulge me with a few more examples, I make no doubt of being soon able to solve any question in this rule.

Philo. I will readily give you more examples, and am pleased to see you are so desirous of overcoming every difficulty that may happen in the course of your practice; and be assured, that assiduity will easily surmount every thing that may appear at first intricate and troublesome.

EXAMPLE II.

What is the cube root of 5735339?

5735339

$$\begin{array}{r}
 \triangle 5735339 \text{ (179)} \\
 \hline
 4735 \text{ Resolvend} \\
 \text{Add } \left\{ \begin{array}{l} 3 \text{ Triple square of the root 1.} \\ 3 \text{ Triple of 1.} \end{array} \right. \\
 \hline
 33 \text{ Divisor.} \\
 \text{Add } \left\{ \begin{array}{l} 147 \text{ Triple of 7, multip. by square of 7.} \\ 21 \text{ Triple of 7, multip. by the root 1.} \\ 343 \text{ Cube of 7.} \end{array} \right. \\
 \hline
 3913 \text{ Subtrahend.} \\
 \hline
 822339 \text{ The new resolvend.} \\
 \text{Add } \left\{ \begin{array}{l} 867 \text{ The triple square of root 17.} \\ 51 \text{ Triple of 17.} \end{array} \right. \\
 \hline
 8721 \text{ Divisor} \\
 7803 \text{ Triple of 17, multiplied by 9.} \\
 4131 \text{ Triple of 17, multip. by square of 9.} \\
 729 \text{ The cube of 9.} \\
 \hline
 822339 \text{ Subtrahend.}
 \end{array}$$

These two examples will be sufficient to direct you in the method of proceeding in the operation: but it may be necessary to add several questions with their answers, leaving the operations as an exercise.

1. What is the cube root of 84604519 ?
Answer 439.
2. What is the cube root of 1092727 ?
Answer 103.
3. What is the cube root of 27054036008 ?
Answer 3002.
4. What is the cube root of 219365327791 ?
Answer 6031.
5. What is the cube root of 122615327232 ?
Answer 4968.

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In all the above questions the given number is a true cube; but when this is not the case, you must annex three cyphers to the right hand of the remainder, and proceed in the operation as before, and the remaining figures in the root will be decimals; as in the following questions.

6. What is the cube root of 25917056 ?

Answer 295.9

7. What is the cube root of 93759.575070 ?

Answer 45.42 .

8. What is the cuberoot of 67507824239 ?

Answer 4071.8 .

When the true cube root of any number cannot be found, you must observe in proving the question, to add the remainder to the cube of the root, and then the sum will be equal to the number given, if the operation be rightly performed.

DIALOGUE VI.

Of DUODECIMALS; Or, the art of multiplying FEET, INCHES, and PARTS; commonly called CROSS MULTIPLICATION.

Discipulus. **W**HY do you call this rule Duodecimals?

Philo. Because the unit or integer, is divided into 12 equal parts. And hence this way of computation is chiefly used among workmen, in casting up the contents of superficial and solid works, the lineary dimensions being generally taken in feet, inches and parts:

BECAUSE,

1 Lineary foot is 12 inches.

1 Inch is 8 or 12 Lineary parts.

Also 1 Square foot is 144 inches.

1 Square inch is 64 or 144 square parts:

And 1 Cubic foot is 1728 cubic inches.

1 Cubic inch 512, or 1728 cubic parts.

N. B. The

N. B. The difference in the parts, arises from considering the inch, as divided into 8 or 12 equal parts.

Discipulus. But might not all this trouble be saved, by taking the linear dimensions in feet, and decimal parts of a foot?

Philo. Certainly it might, and the operation performed with much greater ease and accuracy: But custom has prevailed; and the dimensions are now generally taken in feet, inches, and parts. I say generally, because many take the dimensions in feet and decimal parts: but, as this is not commonly practised, I would not have you ignorant of the usual method of computation, though the contents of any figure may be more easily found by decimals.

Discipulus. I thank you, Sir, for giving me satisfaction in this particular; and am greatly obliged to you for instructing me in the methods commonly practised; as I know it is a very difficult task to convince such persons, that their method is not the best hitherto known.

Philo. In order to remove every difficulty attending this method of multiplying feet, inches, and parts, it will be necessary to observe, that some workmen take their dimensions in feet, inches, and half quarters, or eighth parts: others take them in feet, inches and quarters, reckoning every quarter as three parts; so that these actually follow the duodecimal scale; though they seldom take notice of any other parts of an inch, than 3, 6, or 9, and give and take, as they call it, for the intermediate ones. This being observed, the operation of multiplying feet, inches, &c. is performed by this general

R U L E.

1. Under the multiplicand write the correspondent denominations of the multiplier.
2. Multiply each term in the multiplicand, beginning with the lowest, by the feet in the multiplier; placing each result under its respective term, and remembering to carry an unit for every 12 from each lower denomination to its next superior.
3. Multiply in the same manner, all the multiplicand, by the inches in the multiplier; and write the

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result of each term, one place to the right hand of those in the multiplicand.

4. Work in the same manner with the parts in the multiplier, setting the result of each term removed two places to the right hand of those in the multiplicand.

And the sum of these will give the product required.

EXAMPLE I.

Let it be required to multiply 10 feet, 4 inches, 5 parts, by 7 feet, 8 inches, 6 parts.

OPERATION.

F.	I.	P.	
10	4	5	Multiplicand.
7	8	6	Multiplier.
72	6	11	
6	10	11	4
	5	2	2 : 6
79	11	0	6 : 6 Answer:

The product would have been the same, if the work had been begun with the lowest name in the multiplier; observing to place the results right.

EXAMPLE

	10	4	5	Multiplicand.
	7	8	6	Multiplier.
		5	2	2 : 6
Add {	6	10	11	4
	72	6	11	
	79	11	0	6 : 6 Answer.

Here the answer is the same as before; though the operation is begun in the same manner as in common multiplication.

From this operation carefully observed, the following rule is easily deduced.

RULE.

Feet multiplied by feet, give feet.

Feet multiplied by inches, give inches.

Feet

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Feet multiplied by parts, give parts.

Inches multiplied by parts, give parts.

Parts multiplied by parts, give other parts, each of which is the 144th part of an inch.

I have given you the above example merely that you may not be at a loss how to proceed, if you should have the dimensions given in feet, inches, and parts: but this is very rarely done, the dimensions being thought sufficiently accurate if taken in feet and inches. I shall therefore, in the remaining examples confine myself to these, as being fully sufficient to answer most of the purposes of practical mensuration.

EXAMPLE II.

Let it be required to multiply 7 feet, 10 inches, by 6 feet, 4 inches.

OPERATION.

$$\begin{array}{r}
 7 : 10 \text{ Multiplicand.} \\
 6 : 4 \text{ Multiplier.} \\
 \hline
 \text{Add } \left\{ \begin{array}{r} 47 : 0 \\ 2 : 7 : 4 \\ \hline 49 : 7 : 4 \text{ Answer.} \end{array} \right.
 \end{array}$$

Here I begin with the 6 feet, saying 6 times 10 is 60, which is just 5 feet, (for according to the above rule, feet multiplied by inches give inches) I therefore set 0 under the inches, and carry 5; then 6 times 7 is 42, and 5 is 47, which I set under feet; because feet multiplied by feet give feet. I then proceed to the 4 inches, saying 4 times 10 is 40, 4 and carry 3; I therefore write the 4 in the place of parts, because inches multiplied by inches, give parts: then 4 times 7 is 28, and 3 that I carried is 31 inches, or 2 feet, 7 inches; because feet multiplied by inches, give inches: I therefore place 7 in the column of inches, and 2 in that of feet. These two lines added together give 49 feet, 7 inches, and 4 parts, the answer required.

EXAMPLE

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EXAMPLE. III.

What is the product of 12 feet, 9 inches, multiplied by 10 feet, 6 inches?

OPERATION.

$$\begin{array}{r}
 12 : 9 \text{ Multiplicand.} \\
 10 : 6 \text{ Multiplier.} \\
 \hline
 \text{Add } \left\{ \begin{array}{l} 127 : 6 \\ 6 : 4 : 6 \end{array} \right. \\
 \hline
 133 : 10 : 6 \text{ Answer.}
 \end{array}$$

Here I begin with the 10 feet, saying 10 times 9 is 90 inches, or 7 feet 6 inches; I therefore set down 6 under inches, and say, 10 times 12 is 120, and 7 that I carried is 127, which I set down in the column under feet. Then 6 times 9 is 54 parts, or 4 inches 6 parts, I therefore set down 6 in place of parts; and 6 times 12 is 72, and 4 that I carried is 76 inches, or 6 feet, 4 inches; I therefore set the 4 under the inches, and the 6 under the feet; I then add the two lines together, and the sum 133 feet, 10 inches, 6 parts, is the answer required.

EXAMPLE IV:

Multiply 11 feet 11 inches, by 9 feet 9 inches:

OPERATION.

$$\begin{array}{r}
 11 : 11 \text{ Multiplicand.} \\
 9 : 9 \text{ Multiplier.} \\
 \hline
 \text{Add } \left\{ \begin{array}{l} 107 : 3 \\ 8 : 11 : 3 \end{array} \right. \\
 \hline
 116 : 2 : 3 \text{ Answer.}
 \end{array}$$

EXAMPLE V.

Let it be required to multiply 20 feet 8 inches, by 19 feet 7 inches.

D

OPERATION:

1 INTRODUCTION.

OPERATION.

$$\begin{array}{r}
 20 : 8 \text{ Multiplicand.} \\
 10 : 7 \text{ Multiplier.} \\
 \hline
 \text{Add } \left\{ \begin{array}{l} 206 : 8 \\ 12 : 0 : 8 \end{array} \right. \\
 \hline
 218 : 8 : 8 \text{ Answer.}
 \end{array}$$

These examples wrought at large, will be sufficient, if attentively considered, to explain this rule: I shall therefore only add a few questions with their answers, leaving the operations as an exercise.

EXAMPLES for PRACTICE.

1. What is the product of 97 feet, 8 inches, multiplied by 8 feet, 9 inches?
Answer 854 feet, 7 inches.
2. Multiply 75 feet, 7 inches, by 9 feet, 8 inches.
Answer 730 feet, 7 inches, 8 parts.
3. Multiply 7 feet, 9 inches, by 3 feet 6 inches.
Answer 27 feet, 1 inch, 6 parts.
4. Multiply 75 feet, 7 inches, by 9 feet, 8 inches.
Answer 730 feet, 7 inches, 8 parts.
5. Multiply 87 feet, 5 inches, by 35 feet 8 inches.
Answer 3117 feet, 10 inches, 4 parts.
6. Multiply 259 feet, 2 inches, by 48 feet, 11 inches.
Answer 12677 feet, 6 inches, 10 parts.
7. Multiply 179 feet, 3 inches, by 58 feet, 10 inches.
Answer 10535 feet, 10 inches, 6 parts.
8. Multiply 257 feet, 9 inches, by 39 feet, 11 inches.
Answer 10288 feet, 6 inches, 3 parts.

Discipulus. Your explanations of this rule have been so plain, and the several steps necessary in the operation are so evident, that I sufficiently understand the whole. But I find the operations, unless the numbers are small, to be attended with more trouble, than the same operations by decimal arithmetic; and therefore the rule seems to me of little use.

Philo. You are extremely right, *Discipulus*; and a rule for taking dimensions, decimally divided, would be by far the most easy method of measuring. But custom prevails;

I N T R O D U C T I O N . li

prevails; and when the dimensions are small, this method is attended with very little trouble.

Discipulus. It gives me the highest pleasure to find that I am not wholly mistaken, and I could wish that the decimal method only was used, as it is so very natural, and so properly adapted to every thing of this kind. And would it not be very easy to have a rule decimally divided, so that the dimensions might be taken, without the trouble of reducing the inches, &c. into decimals?

Philo. Extremely easy. The foot may easily be divided into ten equal parts, and each of these parts into ten others: for the divisions will be very little closer than those on the common rule, where the foot is divided into twelve equal parts, and each of these divisions into eight smaller: consequently, the common rule is divided into 96 equal parts; so that the former has only 4 divisions more than the latter. I will add a question, where the same dimensions are taken in the common, and in the decimal manner.

Suppose the length of a board be 18 feet, 6 inches, and is to be multiplied by 6 feet, 6 inches.

OPERATION by the Common Method.

$$\begin{array}{r}
 18 : 6 \\
 6 : 6 \\
 \hline
 111 : 0 \\
 9 : 3 : 0 \\
 \hline
 120 : 3 : 0 \text{ Answer.}
 \end{array}$$

If these dimensions had been taken with a decimal scale, they would have been 18.5 feet, and 9.5 feet: and the operation would have stood thus:

$$\begin{array}{r}
 18.5 \\
 6.5 \\
 \hline
 925 \\
 1110 \\
 \hline
 120.25
 \end{array}$$

Or 120 feet, 3 inches; the same as before. When the dimensions are large, this will be found the most easy method

lii INTRODUCTION.

of solution; as the mind is less burdened, and consequently less subject to mistakes.

I shall conclude this INTRODUCTION with adding a few questions, wherein feet, inches, and parts, are multiplied by feet, inches, and parts; because I would have you very ready in solving questions by this, as well as by the decimal method of computation.

EXAMPLES.

1. Multiply 7 feet, 8 inches, 6 parts, by 10 feet, 4 inches, 6 parts.

Answer 79 feet, 11 inches, 8 parts, 3 second parts.

2. Multiply $368 : 7 : 5$, by $136 : 8 : 4$.

Answer $50756 : 7 : 10 : 9 : 8$.

3. Multiply $7 : 5 : 9$, by $3 : 5 : 3$.

Answer $25 : 8 : 6 : 2 : 3$.

23 OC 62

THE

THE
WHOLE ART
OF
MEASURING
SUPERFICIES and SOLIDS.

BOOK I.
Of MEASURING SUPERFICES:
SECT. I.

DEFINITIONS.

I. **A** POINT has no parts, being nothing more than an assignable place in any line.

II. A line hath only length, and is bounded by two points.

III. A right or strait line, is that which lies even between these points, or is the nearest distance between them.

IV. A Superficies is a figure which hath length and breadth, and is bounded by lines either strait or curve.

V. Parallel lines are those which lie equally distant from one another, in all their parts; and therefore if infinitely extended on the same plane would never meet.

VI. When one line is inclined towards another, whether they are right, or circular, they will, when extended, meet, and form an angle; and the point where they meet, is called the angular point.

VII. When one right line stands upon another, so as not to incline to either side, but makes the angles on each side equal; each of these angles is called a right angle; and the line so standing on the other, is called a perpendicular to that whereon it stands.

VIII. All three-sided figures are called Triangles, but admit of several distinctions: as Equilateral, when the sides are equal: Isosceles, when only two sides are equal: Scalene, when the three sides are unequal; and right-angled, when they have one right angle.

IX. All four-sided figures are called Quadrilaterals, but are divided into Squares, Parallelograms, Rhombus's, and Rhomboides.

A square, is that where all the angles are right, and the sides equal.

A Parallelogram, or oblong square, is a figure that hath all its angles right, and its opposite sides equal.

A Rhombus, is that which hath its four sides equal, but no right angle.

X. A circle, is a plane bounded by one curved line, called the circumference, to which all right lines drawn from a certain point within the figure called its center, are equal.

XI. The diameter of a circle, is a right line drawn through the center, terminated at each end by the circumference; and divides the circle into two equal parts, each of which is called a semi-circle: half the diameter is called the Radius.

XII. The circumference of every circle is divided into 360 equal parts, called degrees; each degree into 60 equal parts, called minutes; and each minute into 60 equal parts, called seconds, &c. Any part of the circumference is called an arch.

XIII. The chord of an arch, is a right line joining the extremities of an arch; and by which the circle is divided into two unequal parts, called Segments.

XIV. A Sector, is a figure comprehended under two Radius's of a circle, and the arch included between those Radius's.

XV. A Polygon is a figure contained under several sides; and called a regular Polygon, if the sides and angles are equal; but if they are not, it is called an irregular Polygon.

A Poly-

A Polygon has different names, according to its number of sides; viz. if it has five sides, it is called a Pentagon; if six, a Hexagon; if seven, a Heptagon; if eight, an Octagon; if nine, a Nonagon; if ten, a Decagon; if eleven, an Undecagon; and if twelve, a Duodecagon.

XVI. The Altitude or height of any figure, is the perpendicular let fall from its summit to its base, or the line on which the figure is supposed to stand.

XVII. The Area of any figure, is the superficial content of it.

SECT. II.

Explanation of the CHARACTERS used in this WORK.

Philo. In order to shorten the work in the several operations used in Measuring, a few characters are made use of, and therefore I think it necessary to explain them here; and beg, my dear Discipulus, that you will peruse them attentively, and make them extremely familiar; for I assure you, they will be found very useful in all kinds of operations.

Discipulus. I shall take particular care to observe your instructions, and shall never consider any thing as a task, that you shall think fit to impose upon me.

Philo. I do not doubt your attention, and shall take care to impose nothing, but what you may easily understand.

+, *More*, the sign of Addition. Wherever this sign is found, it implies, that the number, or quantity following the sign, is to be added to that which precedes it: thus $3+6$, is read 3 *more* 6, and signifies, that 6 is to be added to 3.

—, *Less*, the sign of Subtraction, signifies that the number following it, is to be subtracted from the preceding number: thus $8-5$, is read 8 *less* 5, and signifies, that 5 is to be taken from 8.

×, *Into*, the sign of Multiplication, denotes that the numbers between which this sign is placed, are to be multiplied together: thus 4×8 , is read 4 *into* 8, or 4 multiplied by 8: And $4 \times 5 \times 6 \times 7 \times 8$, implies, that 4 is to be multiplied by 5; that product multiplied

by 6; that result multiplied by 7; and the last product multiplied by 8. See the work.

$$\begin{array}{r}
 4 \\
 \hline
 5 \\
 20 \\
 \hline
 6 \\
 120 \\
 \hline
 7 \\
 840 \\
 \hline
 8 \\
 6720
 \end{array}$$

$\div$, By, the sign of Division, signifies that the number going before the sign, is to be divided by the number following it: thus $20 \div 5$, implies that 20 is to be divided by 5. But the common method now used for expressing division is, by placing the two numbers, in the form of a fraction, the dividend above, and the divisor below it. Thus if 284 were to be divided by 32, it is expressed in the following manner, $\frac{284}{32}$.

$=$, Equal, the sign of Equation, signifies that the numbers, or expressions on each side of it, are equal to one another: thus $6 + 2 = 8$, or 6 more 2 is equal to 8. Also $9 - 3 = 4 + 2$, or 9 less 3 is equal to 4 more 2: and $8 \times 9 = 72$, or 8 multiplied by 9 is equal to 72: also $\frac{15}{3} = 5$, or 15 divided by 3 is equal to 5.

The terms in proportion to one another, are expressed in the following manner: $8 : 12 :: 16 : 24$, that is, 8 is to 12, as 16 is to 24. The two points therefore between the 8 and 12, signify *is to*; the four points between the 12 and 16 imply *as*; and the two points between the 16 and 24 denote *is to*.

$\sqrt{}$, The sign of the square root, implies, that the square root of those numbers or quantities to which it is prefixed, must be extracted: thus $\sqrt{81}$, signifies that the square root of 81, must be extracted. When several numbers or quantities, connected by the other signs or characters, are to have their square root extracted, a line is drawn over them, to denote what part of the expression is to have its square root extracted; as $\sqrt{16 \times 14}$, implies that 16 is to be multiplied by 14, and

and the square root is to be extracted out of the product.

The character $\sqrt[3]{}$ denotes the cube root of the quantities following it: thus, $\sqrt[3]{397}$, implies the cube root of 397: and $\sqrt[3]{32 \times 69}$, signifies, that 32 is to be multiplied by 69, and the cube root extracted out of the product.

When several terms, or numbers, are connected by lines drawn over them, it denotes the result of those terms or numbers, when the operations denoted by the signs are performed: thus $6 - 4 \times 6 + 4$, implies, that 6 *less* 4, or 2 is to be multiplied by 6 *more* 4, or 10: and $8 \times 6 - 4 + 14$, signifies that 8 is to be multiplied by 6 *less* 4 *more* 14, or 16. But had the line been extended only to the first 4, or $8 \times 6 - 4 + 14$, the expression would have been very different; for it would then have signified, that 8 was to be multiplied by 6 *less* 4, or 2, and 14 added to the product: in the first case, the result would have been 124, and in the latter only 30.

Hence appears the necessity of being careful in drawing these lines; as they greatly alter the value of the expression.

These, my dear Discipulus, are all the characters necessary for you to use in any kind of mensuration: and I would advise you to get them perfectly by heart, and practise them frequently, by which means they will soon become as familiar to you, as the common figures.

Discipulus. It shall be my business to imprint them firmly in my memory. Indeed their use in shortening the operations is so apparent, that I should think I profited very little by your instructions, if I neglected to learn what has so great a tendency to promote my own advantage.

Philo. It gives me pleasure to hear you make such just observations: and I sincerely wish all scholars would endeavour to imitate your example; this would at once give satisfaction to their master, and delight to their parents. Give me also leave to point out to you another instance, where these signs are very necessary: I mean in expressing the operations proper to be performed, when you are not in a place convenient for the

purpose. The operations in measuring are generally long, and require a considerable degree of care and attention; and therefore, often very improper to be performed, on the spot where the dimensions are taken. But the whole may be easily expressed by the assistance of these characters, and you will thence be enabled to perform the requisite operations at your leisure. But in order to reap this advantage, it will be proper for you to be very ready in expressing any calculation necessary to be performed; and this you will soon acquire by practice.

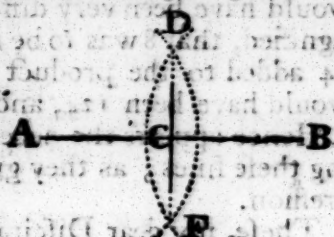
S E C T. III.

Of such GEOMETRICAL PROBLEMS, as are necessary in MENSURATION.

P R O B. I.

To divide a right line, as AB, into two equal parts.

From A and B, the ends of the given line, and any Radius greater than one half of AB, describe two arches of circles crossing each other in two points, as D and F, then join those two points D and F with a right line, and it will bisect, or divide the line AB, into two equal parts, as at C.

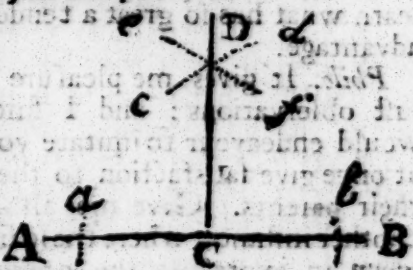


P R O B. II.

To raise a perpendicular on a given right line, from a point assigned, situated in, or near the middle of it.

Let the given line be AB, and let it be required to erect a perpendicular to it, in the point C.

Place one point of your compasses in the point C, and open them to any convenient distance, as to *a*, and make a mark at *b* with the same extent of the compasses. Then placing one point of the compasses in *b*, with any convenient Radius greater than *bC*, describe

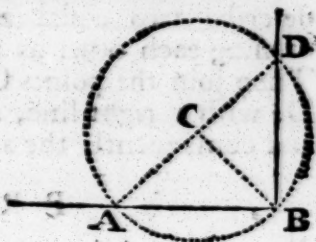


the arch cd , and with the same Radius, placing the point in a , describe the arch ef . Draw the right line CD , from the given point C , through the intersection of the two arches cd, ef , and it will be the perpendicular required.

P R O B. III.

To raise a perpendicular upon the end of any given line.

Let the given line be AB , and let it be required to raise a perpendicular on the point B . Upon any point at random, out of the line AB , as at C , place one foot of your compasses, and extend the other to B , and on C , as a center describe a circle, and from the point where the circle cuts the given line, as at A , draw the circle's diameter ACD : then from the point D , draw the right line DB , and it will be the perpendicular required.



P R O B. IV.

To let fall a perpendicular upon any right line, from any assigned point, that is not in the given line.

Let the given point be C , and the given line on which the perpendicular is to let fall be AB .

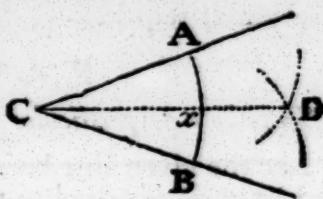
Set one foot of the compasses in the given point C , and describe such an arch of a circle as will cross the given line AB , in two points as D and E . Then setting one foot in D open the compasses to any distance greater than half DE , and describe an arch as at F ; and with the same Radius, setting one point in E , with the other describe another arch, cutting the former in F : Then laying a ruler on F and C , draw the right line CF , which will be the perpendicular desired.



P R O B. V.

To bisect an angle into two equal angles.

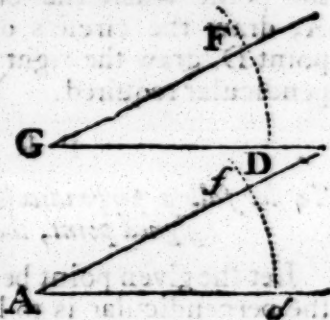
Upon the angular point C as a center, and with any convenient Radius, describe an arch as AB, and from these two points A and B, describe two equal arches crossing each other as at D. Then join the points C and D, with a right line, and it will bisect the arch AB, and consequently the angle, as was required.



P R O B. VI.

From a point A in a given right line, to make a right lined angle, equal to another right lined angle given.

Upon the given angular point C, and with any convenient Radius, describe an arch, as FD; and with the same Radius, setting one point of the compasses in A, describe the arch fd; make Fd equal to FD; then join the points A and f, with a right line, and it will form the angle required.

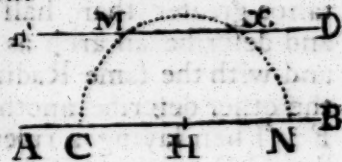


P R O B. VII.

To draw a right line, that shall be parallel to a given right line, and pass through any assigned point.

Let the given right line, be AB, and the point thro' which the parallel is to pass be x.

Take any point in the given line as at H, the farther from x the better; make Hx Radius, and with it upon the point H, describe a semi-circle, as CMN; then make the arch CM equal to the arch N x, and through the points



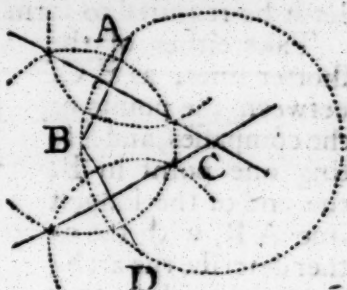
points M and x , draw the right line F D, which will be parallel to A B, as required.

P R O B. VIII.

To describe a circle that shall pass through three given points, provided they are not situated in a strait line.

Let the given points be A, B, and D, thro' which it is required the circumference of the circle shall pass. Join the points B A, and B D with right lines, and bisect them both, as already described, Prob. 1.

The point where these bisecting lines meet, as at C, will be the center of the circle required; for if you place one of the points of the compasses in C, and extend the other to any of the given points A, B, or D, and with that Radius describe a circle it will pass thro' the three given points.

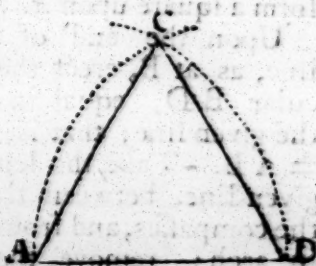


P R O B. IX.

Upon a given right line to describe an equilateral triangle.

Let the given line be A D, on which it is required to describe an equilateral triangle.

Make the given line Radius, and with that distance place one of the points of the compasses on A, and with the other describe the arch C D: remove the point of the compasses to D, and with the same Radius or distance describe the arch A C; from the intersection of these arches draw the right lines A C, and D C, and you will have the equilateral triangle required.

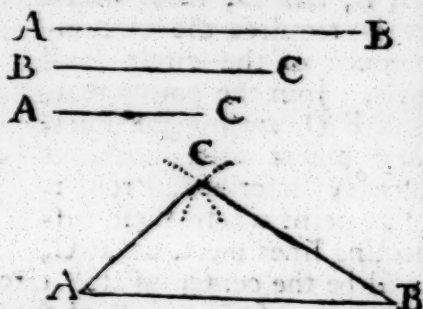


P R O B. X.

Three right lines being given, provided any two of them taken together be longer than the third, to form them into a triangle.

Let the three given lines be AB , BC , and AC , and let it be required to form them into a triangle.

Take either of the shorter lines, as BC , between the points of the compasses, and setting one point in B , the end of the longest line AB , with the other describe the arch; then take the other line, AC , between the points of the compasses, and setting one foot in A , with the other describe a second arch, cutting the former in C ; join the points CA , and CB , with right lines, and you will form the triangle required.



P R O B. XI.

Upon a given right line to form a square.

Let the given line be AB , and let it be required to form a square upon it.

Upon the end of the given line, as at B , erect the perpendicular BD , equal in length to the given line; that is, make $BD = AB$. Take the length of the given line, between the points of the compasses, and from A describe an arch; remove the point to D , and with the same distance, or Radius, describe another arch cutting the former in C ; join the points CA , and CD , with right lines, and they will form the square required.

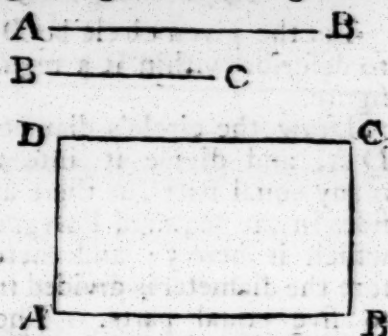


P R O B. XII.

Two right lines being given to form of them a right-angled Parallelogram.

Let the two lines be AB and BC , and let it be required to form of them a right-angled Parallelogram.

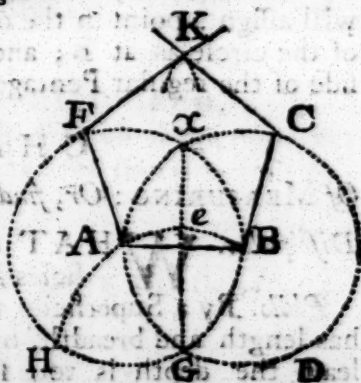
Upon an end of the longest line, as at B ; erect a perpendicular of the same length with the shorter line BC ; then from the point C draw a line parallel to AB , and of an equal length; that is, make $DC = AB$; join AD with a right line, and you will form the Parallelogram required.



P R O B. XIII.

Upon a right line to describe any regular Pentagon, or five sided Figure.

Let the right line be AB , and let it be required to describe on it a regular Pentagon. Make the given line AB Radius, that is, take the length of the line AB , between the points of your compasses, and upon each end of it describe a circle, and upon these points where the circles cross each other, as at G , and H , draw the right line GH . Upon the point G , with the same Radius, describe the arch $HAeBD$; lay a ruler on the points D, e , and mark where it crosses the other circle, as at F . Again, lay the ruler upon the points H, e , and mark where it crosses the other circle, as at C . Then from the points F and C , with the same Radius as before, describe cross arches, as at K : join the points AF, FK, KC , and CB , with right lines, and they will



form the Pentagon required. For $AB=AF=FK=KC=CB$; and the angles at A, B, C, K, F, will be equal.

P R O B. XIV.

In any given Circle to inscribe any regular Polygon.

Let the given circle be D A B, and let it be required to describe within it a regular Pentagon, or five sided figure.

Draw the circle's diameter D A, and divide it into as many equal parts, as there are sides in the required Polygon, which is here 5; and therefore the diameter is divided into five equal parts. Then make the whole diameter Radius, and describe two arches, as C A, and C D. If a right line be drawn from the point C through the second division on the diameter, as at 2, it will assign a point in the opposite part of the periphery of the circle as at B; and the line D B, will be one side of the regular Pentagon required.



C H A P. II.

Of MEASURING: Or, finding the Area of SUPERFICIES.

Discipulus. **W**HAT do you mean by a Superficies?

Philo. By a Superficies we understand a figure, which has length and breadth, but is without thickness: at least the depth is too small to be regarded in this Species of Mensuration: the surface only being here considered.

S E C T. I.

Of Measuring a SQUARE.

A square has already been defined, in the first section of the preceding chapter, and the area of it is found by the following

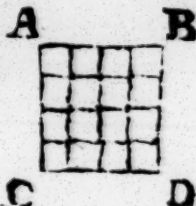
RULE.

RULE.

Multiply the side into itself, and the product will be the area, or content required.

EXAMPLE.

Suppose the figure ABCD a geometrical square, whose side, for they are all equal, is 4 feet; and its area or content be required.



OPERATION.

$$\begin{array}{r} 4 \\ \times 4 \\ \hline 16 \end{array}$$

Answer 16 feet.

You will observe, my dear Discipulus, that nothing more is required, than to find how many small squares the sides of which are each one foot, are contained in the given square; for the number of these squares is the area of the given square, as you will easily perceive by examining the annexed figure, and comparing it with the above rule and operation.

S E C T. H.

Of Measuring a PARALLELOGRAM.

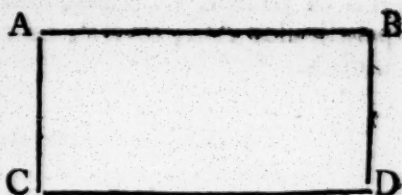
A Parallelogram, is a long square, whose opposite sides are equal and parallel; and its area found by the following

RULE.

Multiply the length by the breadth, and the product will be the area required.

EXAMPLE.

Let ABCD represent a Parallelogram, whose length AB, or CD, is 9 feet 6 inches, and the breadth AC, or BD, 6 feet 3 inches; and it be required to find the area.



Opera-

Operation by Decimals.

$$\begin{array}{r}
 95 \\
 6.25 \\
 \hline
 475 \\
 190 \\
 \hline
 570 \\
 59.375 \\
 12 \\
 \hline
 4.500 \\
 2 \\
 \hline
 1.000
 \end{array}$$

Answer 59 feet, 4 inches and a half.

Operation by Duodecimals, or Cross Multiplication.

$$\begin{array}{r}
 9.6 \\
 6.3 \\
 \hline
 57.0 \\
 2.4.6 \\
 \hline
 59.4.6
 \end{array}$$

Answer 59 feet, 4 inches, and 6 twelfths, or one half, as before.

S E C T. III.

Of Measuring a RHOMBUS.

A Rhombus, or Diamond-like Figure, is that which has four equal sides, but no right angle, and is measured by multiplying one of the sides by a perpendicular, let fall from one of the obtuse angles upon the opposite side.

EXAMPLE.

Let ABDC, be a Rhombus whose area is required; and let the length AB be 20 feet, 8 inches, and the perpendicular AG, or BP, be 17 feet, 3 inches.



Operation

Operation by Decimals.

$$\begin{array}{r}
 20.75 \text{ Length} \\
 17.25 \text{ Perpendicular.} \\
 \hline
 10375 \\
 4150 \\
 14525 \\
 3075 \\
 \hline
 357.9375 \\
 12 \\
 \hline
 11.2500 \\
 8 \\
 \hline
 2.0000
 \end{array}$$

S E C T. IV.

Of Measuring a RHOMBOIDES.

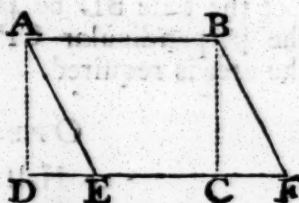
A Rhomboides, is a figure whose opposite sides and opposite angles are equal, but it has no right angle; and is measured by the following

R U L E.

Multiply one of the longest sides, by the perpendicular, let fall from one of the obtuse angles, and the product is the content required.

E X A M P L E.

Let ABFE, be a Rhomboides, its length $AB=EF$, be 20 feet, the perpendicular $AD=BC$, 12 feet, 6 inches, and let the area be required,



Operation by Decimals.

$$\begin{array}{r}
 20 \\
 12.5 \\
 \hline
 100 \\
 40 \\
 20 \\
 \hline
 250.0
 \end{array}$$

Answer 250 feet.

Operation

Operation by Duodecimals.

$$\begin{array}{r} 20 \\ 12 \overline{) 6} \\ \hline 240 \\ 10:0 \\ \hline 250 \ 0 \end{array}$$

Answer 250 feet, as before.

S E C T. V.

Of Measuring a TRIANGLE.

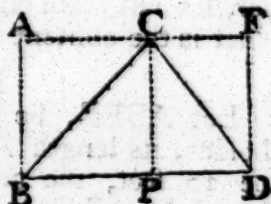
There are various sorts of triangles, but the area of them all are found by this general

R U L E.

Multiply half the base by the perpendicular, or the whole base by half the perpendicular; and the product will be the area required.

E X A M P L E.

Let BCD, be a triangle. From the point C, let fall the line CP, perpendicular to the side BD. Let the base BD be 40 feet, and the perpendicular CP 20 feet: the area is required.



O P E R A T I O N.

$$\begin{array}{r} 20 \text{ Half the Base.} \\ 20 \text{ Perpendicular.} \\ \hline 400 \end{array}$$

Answer 400 feet.

The figure annexed will sufficiently explain the reasons on which the rule is founded: for the triangle BCD is half the Parallelogram AFDB. Therefore the area of the triangle BCD, must be equal to half the area of the Parallelogram; but the area of the Parallelogram, is equal to $BD \times CP = 40 \times 20 = 800$; half of which

which is 400 feet, the area of the triangle BCD; and the same as before.

But the area of a triangle, when the three sides are given, may be found without letting fall a perpendicular, by the following

R U L E.

From half the sum of the three sides, subtract each side separately; let the half sum, and the three differences be multiplied continually, and the square root of the product will be the content required.

E X A M P L E.

Suppose the area of a triangle was required, whose sides were 20, 18, and 16 feet.

O P E R A T I O N.

The sum of the three sides 20, 18, and 16, is 54, whose half is 27.

And $27 - 20 = 7$; $27 - 18 = 9$; $27 - 16 = 11$. Then

$$\begin{array}{r}
 27 \\
 \underline{7} \\
 189 \\
 \underline{9} \\
 1701 \\
 \underline{11} \\
 18711
 \end{array}$$

18711 (136.78

$$\begin{array}{r}
 1 \\
 23 \overline{) 87} \\
 \underline{31} \\
 266 \\
 \underline{61} \\
 2727 \\
 \underline{71} \\
 27348 \\
 \underline{81} \\
 22316
 \end{array}$$

Answer 136.78 feet.

SECT

THE WHOLE ART.

S E C T. VI.

Of Measuring a TRAPEZIUM.

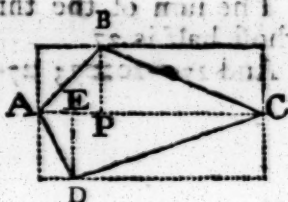
A Trapezium is a quadrangular figure, whose opposite sides are not parallel; and its area, or superficial content, is found by the following

R U L E.

Divide the Trapezium into two triangles, by drawing a Diagonal through the figure; let fall perpendiculars from the opposite obtuse angles; take the half of the sum of the two perpendiculars, and multiply the Diagonal by that half sum; the product will be the area required.

E X A M P L E.

Let ABCD be a Trapezium, whose area, or superficial content, is required; supposing the Diagonal AC to be 200 feet, the perpendicular BP 50 feet, and the perpendicular DE, 40 feet.



O P E R A T I O N.

BP 50 + DE 40 = 90, the half of which is 45.

Then 200 = The Diagonal A C.

45 = Half the perpendiculars.

1000

800

9000 Feet, the area of the Trapezium

ABCD, required.

N. B. The area would have been the same, if the area of each triangle had been found separate, and their areas added together,

200 The Diagonal.
 25 Half BF.

1000.

400

5000 Area of ABC.

200 The Diagonal.

20 Half DE.

4000 Area of ADC.

Then $5000 + 4000 = 9000$, the area of the whole Trapezium ABCD, the same as before.

S E C T. VI.

Of Measuring irregular FIGURES.

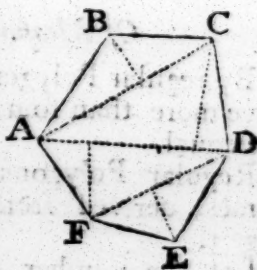
Irregular Figures are such as have more sides than four, and both the sides and angles unequal; their area, therefore, is found by the following

R U L E.

Divide the irregular figures into Trapeziums and triangles; find the area of each severally, and add all the areas together: the sum will be the area of the irregular figure.

E X A M P L E.

Let ABCDEF, be an irregular figure, whose area is required. Draw the lines AC, AD, and DF, which will divide the figure into four triangles; and let fall the perpendiculars from the angles, at B, C, E, and F: and let AC be = 40, AD = 30, and DF = 26 feet; and the perpendiculars B = 14, C = 28, E = 15, F = 16.



OPERATION.

THE WHOLE ART

OPERATION.

$$40 = AC.$$

$$\begin{array}{r} 7 = \text{Half perp. B.} \\ \hline 280 = \text{Area of ABC.} \end{array}$$

$$30 = AD$$

$$\begin{array}{r} 14 = \text{Half perp. C.} \\ \hline 420 = \text{Area of ACD.} \end{array}$$

$$30 = AD.$$

$$\begin{array}{r} 8 = \text{Half perp. F.} \\ \hline 240 = \text{Area of ADF.} \end{array}$$

$$26 = DF.$$

$$\begin{array}{r} 7.5 = \text{Half perp. E.} \\ \hline \end{array}$$

$$130$$

$$182$$

$$\begin{array}{r} \hline 195.0 = \text{Area of DEF.} \end{array}$$

Then $280 = \text{Area of ABC.}$

$$420 = \text{Area of ACD.}$$

$$240 = \text{Area of ADF.}$$

$$195 = \text{Area of DEF.}$$

$$\begin{array}{r} \hline 1135 = \text{The Area of ABCDEF.} \end{array}$$

S E C T. VII.

Of Measuring regular POLYGONS.

By regular Polygons, are meant all such figures, as have more than four sides, and whose sides and angles are equal.

Regular Polygons are distinguished by particular names, derived from the Greek, as is explained in page 3.

Let the number of sides and angles be what they will, the area or superficial content is found by the following

RULE,

RULE.

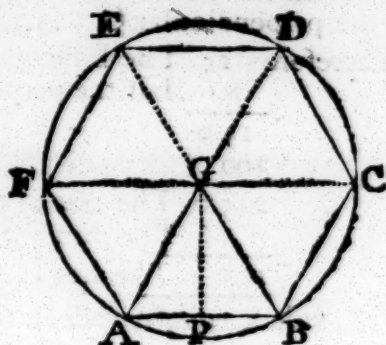
Multiply the sum of the sides by half the perpendicular let fall from the center to the middle of one of the sides: or, multiply half the sum of the sides by the perpendicular, and the product will be the content.

EXAMPLE.

Let it be required to find the area of a regular Hexagon, whose side is twenty-five yards, and the perpendicular 17.2 yards.

SOLUTION.

The figure annexed, represents a regular Hexagon, or six sided figure inscribed in a circle; the six sides are the strait lines AB, BC, CD, DE, EF, and FA. And GP, is the perpendicular let fall from the center G, upon the middle of the side AB.



Then $25 \times 6 = 150 =$ to the sum of the sides; and half the perpendicular $= 8.6$. Therefore

$$\begin{array}{r}
 150 \\
 8.6 \\
 \hline
 900 \\
 1200 \\
 \hline
 1290.0 \text{ The area required:}
 \end{array}$$

Or, 75 Half the sum of the sides
 17.2 The perpendicular.

$$\begin{array}{r}
 150 \\
 525 \\
 75 \\
 \hline
 1290.0 \text{ The area required;}
 \end{array}$$

THE WHOLE ART

Or, 150 The sum of the sides.
 17.2 The Perpendicular.

$$\begin{array}{r}
 300 \\
 1050 \\
 150 \\
 \hline
 2|25800|1290.0 \text{ The area required.}
 \end{array}$$

It is sufficiently evident from the above figure, that the Hexagon consists of six triangles, all equal to one another. If therefore the area of one of the triangles be found, and that area be multiplied by the number of triangles, we shall have the area of the whole figure.

Thus in the above figure, the side AB is 25 yards, and the perpendicular GP 17.2 yards;

Therefore 25 The Base.
 8.6 Half the perpendicular.

$$\begin{array}{r}
 15.0 \\
 200 \\
 \hline
 215.0 \text{ The area of the triangle AGB.} \\
 6 \text{ The number of triangles.} \\
 \hline
 1290.0 \text{ The area of the Hexagon, as before.}
 \end{array}$$

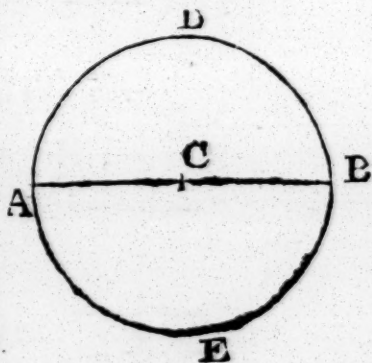
This rule is equally true in all regular Polygons.

S E C T. VIII.

Of Measuring CIRCLES, and their PARTS.

A circle is a plane figure, comprehended under one line only, called the circumference, or Periphery.

In the annexed figure, ADBE is the circumference of the circle ABCDE. C, the center, and AB the diameter, half of which, viz. AC, or CB, is called the Radius.



The circle is the most capacious figure, or contains more space than any other plane figure of equal compass: and at the same time, affords a greater number of useful properties, most of

of which depend upon knowing the proportion, which the diameter bears to the circumference.

It is sufficiently evident from what we have observed in the preceding section, that a circle may be considered as a Polygon, of an infinite number of sides; that the Radius, or semi-diameter, must be equal to the perpendicular of such a Polygon; and that the circumference must be equal to the Periphery of the Polygon.

But the labour and difficulty attending this method of finding the sides of such a Polygon, has induced Mathematicians to attempt the solution of this useful Problem, by more easy and expeditious methods, and they have so well succeeded, that the calculation is attended with very little difficulty; but the principles on which the modern methods are founded, are of too subtle a nature, to be explained in this work: and therefore, my dear Discipulus, you must be satisfied with knowing the foundation, on which the ancients proceeded. Perhaps hereafter, the modern improvements may be explained.

The famous VAN CULLEN, by pursuing the ancient method, found, with infinite labour, the circle's circumference to 36 places of figures, viz. if the diameter of a circle be 1, the circumference will be 3.14159265358979323846264338327950288. These figures were said to be engraven on his tomb-stone, in St. Peter's church, at Leyden, as a memorial of so great a work. But since his time, Mr. JOHN MACHIN, secretary to the Royal Society carried it to an hundred places in decimals, and demonstrated, that if the circle's diameter be 1, the circumference will be 3.141592653589793238462643383279502884197169399375105820974944592307816406286208998628034825342117067,9. This amazing accuracy has rendered all attempts since superfluous.

But as a very small degree of accuracy is sufficient in common practice, the proportion of ARCHIMEDES, viz. as 7 to 22; or, that of METIUS, as 113 to 355 is commonly used; though that of VAN CULLEN, viz. as 1, to 3.1416 seems better adapted to Mensuration, as it saves the trouble of division.

THE WHOLE ART

PROB. I.

To find the circumference of a Circle, the diameter being given.

By the assistance of either of the above proportions, the circumference of the circle is easily found, when the diameter is given.

EXAMPLE.

Let it be required to find the circumference of a circle, whose diameter is 120 feet, yards, &c.

1 By ARCHIMEDES' proportion.

As 7 : 22 :: 120

$$\begin{array}{r}
 22 \\
 \hline
 240 \\
 240 \\
 \hline
 7 \overline{) 2640} \quad (377.1428, \&c. \quad \text{The circumference required.} \\
 \underline{21} \\
 54 \\
 \underline{49} \\
 \cdot 50 \\
 \underline{49} \\
 \cdot 10 \\
 \underline{7} \\
 30 \\
 \underline{28} \\
 20 \\
 \underline{14} \\
 60 \\
 \underline{56} \\
 \cdot 4
 \end{array}$$

2. By

OF MEASURING.

23

2. By METIUS' proportion.

As 113 : 355 : : 120

$$\begin{array}{r}
 120 \\
 \hline
 7100 \\
 355 \\
 \hline
 113) 42600 \text{ (376.9911, \&c. The circumference)} \\
 \underline{339} \\
 870 \\
 \underline{791} \\
 790 \\
 \underline{678} \\
 1120 \\
 \underline{1017} \\
 1030 \\
 \underline{1017} \\
 130 \\
 \underline{113} \\
 170 \\
 \underline{113} \\
 57
 \end{array}$$

3. By VAN CULLEN's proportion.

As 1 : 3.1416 : : 120

$$\begin{array}{r}
 120 \\
 \hline
 628320 \\
 \underline{31416} \\
 376.9920 \text{ The circumference required.}
 \end{array}$$

By either of these methods, my dear Discipulus, the circumference of the circle may be found sufficiently near the truth, for common practice: I say sufficiently near the truth, because it is impossible to solve the Problem absolutely accurate.

Discipulus. I perceive, Sir, that the two last are so nearly alike, that either of them may be used, as they give nearly the same result; but there is a considerable difference between them and the first; and therefore, I should reject it, unless it be the most accurate.

Philo. It has no claim to preference on that account: either of the latter giving the answer nearer the truth. It is, indeed, much easier than the second; but the third is preferable, both on account of its ease and accuracy.

PROB. II.

To find the area of the circle, the diameter and circumference being given.

The area of any circle is equal to the area of a Triangle, whose base is equal to the circumference, and its perpendicular equal to the Radius, or half the diameter. And hence the following

RULE.

Multiply half the circumference, by half the diameter; the product will be the area required.

EXAMPLE.

Suppose the diameter be 120, and the circumference 376.992, what is its area, or superficial content?

OPERATION.

$$\begin{array}{r}
 188.496 \text{ Half the circumference.} \\
 60 \text{ Half the diameter.} \\
 \hline
 11309.760 \text{ The area required.}
 \end{array}$$

The area of the circle may also be found, by multiplying the square of the diameter, by 0.7854, the one fourth of the Periphery, when the diameter is 1.

EXAMPLE.

Required the area of a circle, whose diameter is 120.

OPERATION.

OPERATION.

$$\begin{array}{r}
 14400 \text{ Square of the diameter.} \\
 \times 7854 \text{ Common multiplier.} \\
 \hline
 57600 \\
 72000 \\
 115200 \\
 100800 \\
 \hline
 11309.7600 \text{ The area required.}
 \end{array}$$

PROB. III.

The diameter of a circle being given, to find the side of a square, whose area shall be equal to the area of the circle.

All circles are to each other, as the squares of their diameters. But when the diameter of a circle is 1, its area is .7854. And hence we have the following

RULE.

As 1, the square of 1, is to .7854, so is the square of the given diameter, to the area of the given circle, the square root of which, is the side of the square required.

EXAMPLE.

Suppose the diameter of the given circle be 12, what is the side of the equal square?

OPERATION.

As 1. to .7854, so is 144, the square of 12, to the area.

$$\begin{array}{r}
 144 \\
 \times 7854 \\
 \hline
 31416 \\
 31416 \\
 7854 \\
 \hline
 113.0976 \text{ The area of the circle.}
 \end{array}$$

The square root of which is the side of the square required.

Thus 113.0976 (10.634. The side of the square required.

$$\begin{array}{r}
 206 \overline{) 11309} \\
 \underline{6 } \\
 2123 \\
 \underline{3 } \\
 21264 \\
 \underline{4 } \\
 15644
 \end{array}$$

Or, if you multiply the diameter by 0.8862, half the square root of the circumference, when the diameter is 1, the product will be the area required.

Thus 0.8862

$$\begin{array}{r}
 12 \text{ The diameter.} \\
 \hline
 10.6344 \text{ The side of the square as before.}
 \end{array}$$

P R O B. IV.

The side of a square being given, to find the diameter of a circle, whose area is equal to the area of the square.

This Problem is the converse of the former; and therefore, if the side of the given square be multiplied by 1.12837, the product will be the diameter required.

E X A M P L E.

What is the diameter of a circle, when the side of the equal square is 10.6344.

OPERATION.

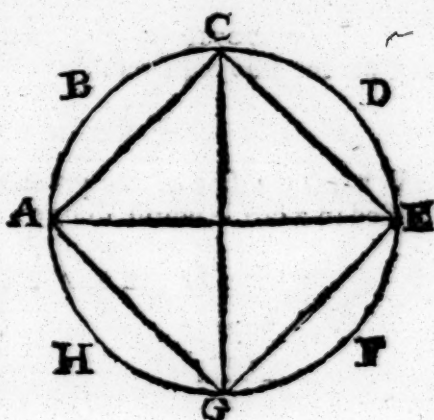
OPERATION.

$$\begin{array}{r}
 10.6344 \text{ Side of the square.} \\
 \underline{1.1283} \\
 319032 \\
 850752 \\
 212688 \\
 106344 \\
 \underline{106344} \\
 11.99879152 \text{ The diameter required, nearly} \\
 \text{equal to 12.}
 \end{array}$$

PROB. V.

The circumference of a circle being given, to find the side of the greatest square that can be inscribed in that circle.

If the circumference of a circle be 1, the side of the greatest square, that can be inscribed within it, is .2251: That is, if the circumference of the circle ABCDEFGH, be 1, the side $AC = CE = EG = GA$, of the greatest inscribed square ACEG, will be .2251. Hence the following.



RULE.

Multiply .2251, by the circumference, and the product is the side of the square required.

EXAMPLE.

Suppose the circumference of the circle be 26, what is the side of the greatest square that can be inscribed in it?

THE WHOLE ART

OPERATION.

$$\begin{array}{r}
 .2251 \\
 \underline{26} \\
 13506 \\
 \underline{4502} \\
 5.8526
 \end{array}$$

The side of the square required.

P R O B. VI.

The diameter of a circle being given, to find the side of the greatest square that can be inscribed in that circle.

If the diameter of a circle be 1, the side of the greatest inscribed square will be .7071 : And hence this general

R U L E.

Multiply .7071, by the diameter, and the product will be the side required.

E X A M P L E.

What is the side of the greatest square that can be inscribed in a circle whose diameter is 140 ?

O P E R A T I O N.

$$\begin{array}{r}
 .7071 \\
 \underline{140} \\
 282840 \\
 \underline{7071} \\
 98.9940
 \end{array}$$

The side of the square required.

Before we proceed farther in solving the Problems, with regard to the circle, I shall here, my dear Discipulus, explain several terms necessary to be understood, before you can proceed with ease and pleasure. For it is my intention to render every thing familiar, and imprint the principles on which each rule is founded, firmly in your memory.

In the annexed scheme
of the circle AEFBHI,
AB is the diameter.
 $AC=CB=CE=CI$,
the Radius or semi-dia-
meter.

$AEC=AIC=BCE=$
 BCI , a quadrant, or a
quarter of the circle.

FCB, a Sector of the
circle.

FB, the Chord of the
arch FbB .

CD, the Secant of the
arch FbB .

BD, the Tangent of
the arch FbB .

FG, the Sine of the
arch FbB .

GB, the versed Sine of the Arch FbB .

FGH, the Chord of the arch $FbBH$.

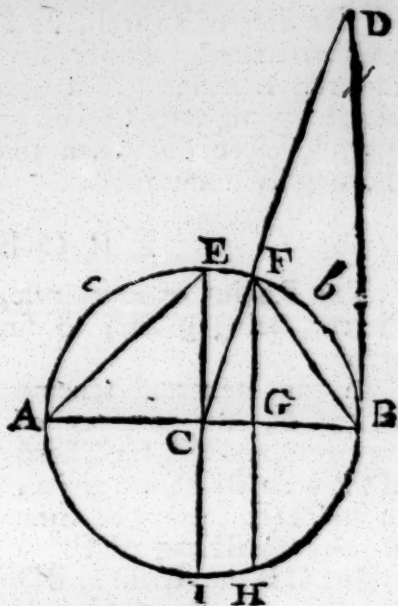
AE, the chord of the arch AcE .

These definitions, my dear Discipulus, I beg you will
imprint carefully in your memory; as I shall often men-
tion them when I come to measuring heights and dis-
tances.

Discipulus. I should very ill deserve the trouble you
have taken on my account, if I should neglect any thing
you desire; and therefore give the closest attention to
your instructions in every particular.

Philo. I well know that you give the utmost atten-
tion to every precept; but I thought it necessary to give
you this caution, because it will save a great deal of
trouble, when it is absolutely necessary to use these
terms. But there is something more than a bare ex-
planation of terms to be learned from this scheme.
The nature and properties of the lines themselves should
be considered: thus for instance,

The Radius, the Tangent, and the Secant, form a
right-angled triangle, and if the Radius of the circle
and the arch be given, both the Secant and Tangent
may be easily found.



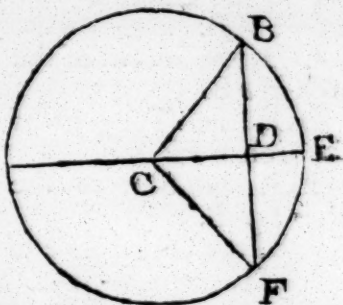
The Sine of an arch, its Chord and versed Sine form another triangle, whose sides increase in proportion as the arch increases. There are several other particulars, which a little attention of your own mind will readily suggest, especially when the following Problems are thoroughly understood.

P R O B. VII.

The Radius of any circle, and the Chord of an arch thereof, being given; to find the versed Sine of that arch.

In the annexed scheme $CE=CB$, the Radius of the circle, and BF , the Chord of the arch BEF , are given; to find DE , the versed Sine of the arch $BE=\frac{1}{2} BEF$.

But CB the Radius, $BD=\frac{1}{2} BF$, the given Chord, and CD , part of the Radius, form a right-angled triangle, in which the Hypotenuse CB , and the perpendicular BD , are given to find CD ; which may easily be done by the 47th proposition of EUCLID's first book, where it is demonstrated, that the square of the Hypotenuse of any right-angled triangle, is equal to the square of the perpendicular, added to the square of the base: and, consequently, if the square of the perpendicular be subtracted from the square of the Hypotenuse, the remainder will be the square of the base; the square root of which being extracted, will give the base required; and the base CD , being subtracted from the Radius CE , will give DE , the versed Sine of EE , or half the arch BEF .



EXAMPLE.

Suppose the Radius $CB = CE$, be 170, and the Chord BF 180, required the versed Sine DE .

OPERA-

O P E R A T I O N.

$170 \times 170 = 28900$ Square of the hypotenuse CB:

And $90 = BD =$ Half the chord BF.

$\frac{90}{8100}$ Square of the perpendicular BD.

Then $28900 - 8100 = 20800$ the square of the base CD.

$$\begin{array}{r}
 \dots \\
 20800 \text{ (144,2} \\
 \underline{1} \\
 24 \mid 108 \\
 \underline{96} \\
 284 \mid 1200 \\
 \underline{1136} \\
 2882 \mid 6400 \\
 \underline{5764} \\
 636
 \end{array}$$

And the Radius $170 - 144.2 = 25.8$ the versed Sine DE required.

P R O B. VIII.

The Radius of a circle, and the versed Sine of an arch being given, to find the Chord of twice that arch.

This is only another part of the above problem, the Radius, or Hypotenuse, and base of the same right-angled triangle are given, to find the perpendicular, the double of which is the Chord required.

R U L E.

From the Radius take the versed Sine, the remainder will be the base of the triangle: Then take the square of the base, from the square of the Hypotenuse, and the remainder with the square of the perpendicular, the square root of which being extracted, and multiplied by 2, will be the Chord required.

E X A M P L E.

EXAMPLE.

Suppose the Radius be 170, and the verfed Sine 26.2. required the Chord of the double arch.

OPERATION.

170—26.2=143.8, The base.

$$\begin{array}{r} 170 \\ 170 \\ \hline 11900 \\ 170 \\ \hline \end{array}$$

28900 Square of the Hypothenufe.

$$\begin{array}{r} 143.8 \\ 143.8 \\ \hline 11504 \\ 4314 \\ 5752 \\ 1438 \\ \hline \end{array}$$

20678.44 Square of the base.

Then $28900 - 20678.44 = 8221.56$ The square of the perpendicular.

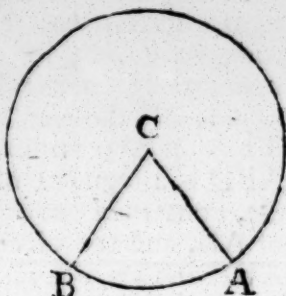
$$\begin{array}{r} \dots \\ 8221.56 \text{ (90.67 The perpendicular, which} \\ 81 \text{ being doubled, is 181.34,} \\ \hline 1806 \mid 12156 \text{ the Chord required.} \\ 10836 \\ \hline 18121 \mid 132000 \\ 126889 \\ \hline \therefore 5111 \end{array}$$

P R O B. IX.

To find the area of a circular Sector, the Radius and length of the sectoral arch being given.

A Sector

A Sector of a circle, is a figure comprehended under two Radii, or semi-diameters, and part of the circular arch: thus in the figure annexed, CBA is a Sector, comprehended under the two Radii, CB, and CA, and the arch, BA, and the area of it is found by the following



RULE.

Multiply the Radius by half the length of the circular arch, and the product is the area or superficial content required.

EXAMPLE.

Suppose the Radius $CB=CA$, be 20 feet, and the length of the arch 24 feet; what is the area of the Sector CBA.

OPERATION.

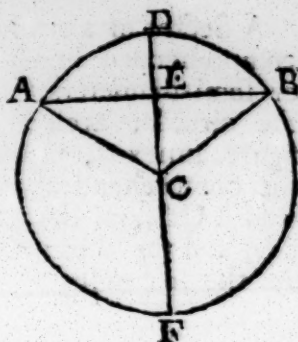
$$\begin{array}{r}
 20 \text{ The Radius.} \\
 12 \text{ Half the arch.} \\
 \hline
 240 \text{ The area required.}
 \end{array}$$

This Rule is founded upon that already given for finding the area of a circle, when the diameter, and consequently the circumference, are given; viz. to multiply half the circumference, by the Radius or half the diameter.

PROB. X.

To find the area or superficial content of any Segment of a circle, when the height and length of the chord of its arch are given.

A Segment of a circle, is a figure terminated by a right line, or chord, less than the diameter of the circle, and by a part of the circumference. Thus in the annexed figure, ABD is a segment of that circle, contained under the Chord of AB, and the circumference ADB and its area may be found by the following



R U L E.

Multiply the height of the Segment by 0.626; add the square of the product to the square of half the Chord: multiply twice the square root of the above sum by two thirds of the height, and the product will be the area required.

E X A M P L E.

Suppose ED, the height of the circular Segment, be 11 feet, and AB, the Chord of the arch, be 37, what is the area of the Segment ABD.

O P E R A T I O N.

$$\begin{array}{r}
 18.5 \\
 18.5 \\
 \hline
 925 \\
 1480 \\
 185 \\
 \hline
 342.25 \text{ Square of half the Chord.}
 \end{array}$$

0.626

II Height of the Segment

$$\begin{array}{r}
 6.886 \\
 6.886 \\
 \hline
 41316 \\
 55088 \\
 55088 \\
 41316 \\
 \hline
 47.416996 \\
 342.25 \\
 \hline
 389.666996 \text{ (19.988.}
 \end{array}$$

$$\begin{array}{r}
 \text{I} \\
 29 \overline{) 289} \\
 \underline{261} \\
 299 \overline{) 2866} \\
 \underline{2601} \\
 2988 \overline{) 26569} \\
 \underline{23904} \\
 29968 \overline{) 266596} \\
 \underline{239744} \\
 26852
 \end{array}$$

19.988

7.32 Two thirds of the height.

$$\begin{array}{r}
 39976 \\
 99964 \\
 \hline
 139916
 \end{array}$$

146.31216 The area required.

This Problem may also be solved in a very different manner, viz. by finding the semi-diameter of the circle $DC = to CB = CA$. This being done, the area of the Sector $CABD$ will be very easily found; and also the area of the triangle ABC : Then if the area of the triangle ABC be subtracted out of the area of the Sector, the remainder will be the area of the Segment ADB .

By the thirtieth proposition of EUCLID's sixth book, half the Chord of any arch is a mean proportional, between

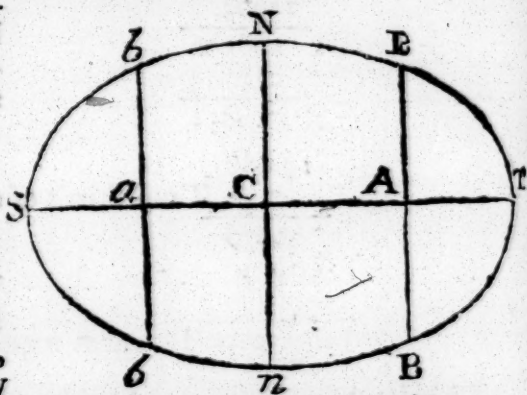
tween the versed Sine, and the remaining part of the diameter. If therefore you square AE , or EB , half the Chord AB , and divide that square by the versed Sine ED , the quotient will be EF , or the part of the diameter wanting; this added to the versed Sine ED , will give the whole diameter DF , the half of which will be the semi-diameter $CD=CA=CB$ required. But the former rule seems preferable; tho' both will give the same result.

SECT. IX.

Of Measuring an ELLIPSIS.

An Ellipsis, vulgarly called an Oval, is a figure bounded by a regular curve line, returning into itself, and formed not from one, but from two centers, called the Foci of the Ellipsis. Its two diameters are unequal, the longer of which is called the transverse diameter, and the shorter the conjugate diameter.

Thus the adjacent figure SN TBC is an Ellipsis; ST , the transverse diameter: Nn , the conjugate diameter; bb , BB , two ordinates; a , A , the two Foci of the Ellipsis: and by their assistance, the Periphery may be drawn in a manner that will be shewn hereafter.



If the transverse diameter ST be intersected, or divided into equal parts, by an ordinate rightly applied, that is, drawn parallel to the conjugate diameter, as at the the points a , C , A , &c: Then are those parts TA , TC , Ta , and SA , SC , SA , usually called Abscissæ, or parts cut off; and the Rectangle of any two parts, being added together, will be equal to the transverse diameter: that is,

TA

$TA + SA = TS$, and $TC + SC = TS$:

Or, $Ta + Sa = TS$, &c.

Every Ellipsis is proportioned, and all such lines as relate to it are regulated, by the following

THEOREM.

As the Rectangle of any two Abscissæ in general : is to the square of half the ordinate which divides them : : so is the Rectangle of any two particular Abscissæ : to the square of half that ordinate which divides them.

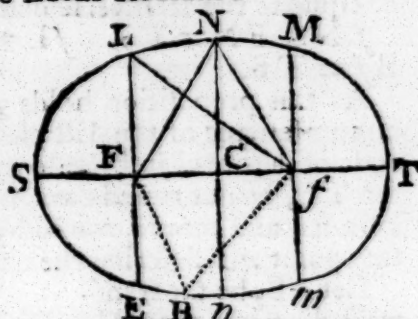
There are several methods used by authors, for finding the Latus Rectum, or third principal line in the Ellipsis, but I think none so easy as the following

THEOREM.

As the transverse diameter : is to the conjugate : : so is the conjugate : to the Latus Rectum :

That is, as $TS : Nn$: : Nn : to LE , the Latus Rectum.

Hence it is evident that LE thus found, is that ordinate by which the other ordinates may be regulated, and discovered: therefore it is the Latus Rectum.



Hence it follows, that if the transverse and conjugate diameters of any Ellipsis are given either in lines or numbers, the Latus Rectum may be easily found ; and, consequently, any ordinate, whose distance from the conjugate diameter is given.

The focus of any Ellipsis, is the distance of the Latus Rectum from the conjugate diameter, or middle of the Ellipsis ; and that distance is always a proportional between the half sum, and the half difference of the transverse and conjugate diameters. And hence this general

THEOREM.

From the square of half the transverse diameter, subtract the square of half the conjugate ; the square root of

of their difference, will be the distance of each focus from the middle or common center of the Ellipsis.

That is, supposing the points f and F to be the true Foci, viz. $FC = Cf$, and $TC = \frac{1}{2} TS$, $NC = \frac{1}{2} Nn$. Then as

$TC + NC : fC :: FC : TC - NC$. Therefore the square of $FC =$ the square of $TC -$ the square of NC . Consequently $FC = \sqrt{TC^2 - NC^2}$. And hence is derived that noble proposition on which both the usual method of describing an Ellipsis, and drawing Tangents is founded.

PROPOSITION.

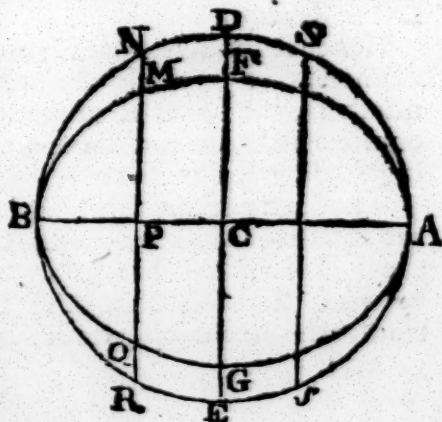
If from the two Foci of an Ellipsis, there be drawn two right lines, so as to meet each other in any point of the Periphery of the Ellipsis, the sum of these lines will be equal to the transverse diameter: that is

$fN + FN = TS$. $fL + FL = TS$. Or, $FB + Bf = TS$.

As this proposition holds good for every point in the circumference of the Ellipsis, it is evident, that if the ends of a thread just the length of the transverse diameter TS , be put round two pins, fixed in the Foci of the Ellipsis, and moved round by a point in the extremity, that point will describe the true Periphery of the Ellipsis.

Let AB , be the transverse diameter of an Ellipsis $AFBG$; GF its conjugate diameter, and $ADBE$, its circumscribing circle: then,

Because in the circle as $AC \times CB : AP \times BP ::$ the square of $CD : \text{the square of } PN$; and in the Ellipsis $AC \times CB : AP \times BP ::$ the square of $CF : \text{the square of } PM$; therefore it will be as, $CD : CF : PN : PM$; or as $AB : FG : NR : \text{to } MQ$. But all the NR 's, SS 's compose the area of the



the circle, and the same number of MQ's form that of the Ellipsis. Hence as $AB : FG$, or the longer diameter of the Ellipsis to the shorter, so is the area of the circumscribing circle, to the area of the inscribed Ellipsis. Wherefore as the square of the diameter of the circumscribing circle, is to the Rectangle of the transverse and conjugate diameters of the Ellipsis; so is the area of the circle, to the area of the Ellipsis.

Hence as the circle is to its circumscribed square, or to the square of its diameter; so is the Ellipsis to its circumscribed Parallelogram, or to the Rectangle of its two diameters.

Hence the area of an Ellipsis may be easily found: for the square root of the Rectangle of its two diameters, will give the diameter of a circle, whose area is equal to the area of the Ellipsis.

Hence, also, as the area of any Segment of the Ellipsis, is to the area of the corresponding circular Segment, so is the conjugate diameter of the Ellipsis, to its transverse.

Hence the area of any Ellipsis is found by these general rules.

R U L E I.

Multiply the transverse diameter, by the conjugate, and that product by .7854, the area of a circle, whose diameter is 1; the last product will be the area required.

E X A M P L E.

Suppose the transverse, or longest diameter be 40.5, and the conjugate or shortest 30.4; required the area of the Ellipsis.

THE WHOLE ART

OPERATION.

$$\begin{array}{r}
 40.5 \\
 30.4 \\
 \hline
 1620 \\
 12150 \\
 \hline
 1231.20 \\
 .7854 \\
 \hline
 492480 \\
 615600 \\
 984960 \\
 861840 \\
 \hline
 966.984180 \text{ The area required.}
 \end{array}$$

RULE II.

Extract the square root, out of the Rectangle of the transverse and conjugate diameters, and the result will be the diameter of a circle, whose area is equal to the area of the Ellipsis.

EXAMPLE.

Suppose the transverse diameter be 40.5. and the conjugate 30.4; what is the area?

OPERATION.

$$\begin{array}{r}
 40.5 \\
 30.4 \\
 \hline
 1620 \\
 12150 \\
 \hline
 \end{array}$$

1231.20

1231.20 (35.088 The diameter required.

$$\begin{array}{r}
 9 \\
 \hline
 65 \overline{) 331} \\
 \underline{325} \\
 7008 \overline{) 62000} \\
 \underline{56064} \\
 70168 \overline{) 593600} \\
 \underline{561344} \\
 32256
 \end{array}$$

But the circumference of a circle, whose diameter is 1, is 3.1416; consequently the circumference of the given circle is 3.1416×35.088 , or 110.232, &c. the half of which is 55.116: consequently.

55.116 Half the circumference.

17.544 Half the diameter.

$$\begin{array}{r}
 220464 \\
 220464 \\
 275580 \\
 385792 \\
 55116 \\
 \hline
 966.935104 \text{ The area required, nearly} \\
 \text{the same as before.}
 \end{array}$$

S E C T. X.

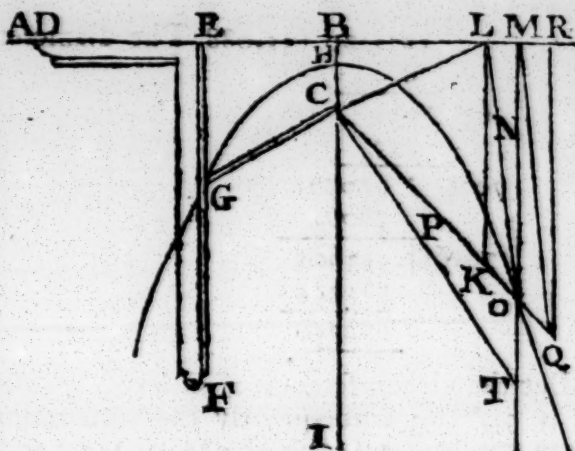
Of Measuring a PARABOLA.

A Parabola is a curvilinear figure, made by the section of a Cone, cut by a Plane parallel to by one of its sides.

In order to explain this definition, which will be clearly understood, when I come to shew the manner of finding the solidity of a Cone, I will shew a method of describing it in Plano.

Draw

Draw a right line as AB , and assume a point C , without it; then in the same Plane, with this line and point, place a square rule DEF , so that the side DE may be applied to the



right line AB , and the other FE turned to the side on which the point C , is situated. This done, and the thread FGC , exactly the length of the side of the rule FE , being fixed at one end of the extremity of the square, as at F , and at the other to the point C , if you slide the side of the rule DE along the right line AL , and by means of a pin, G , continually apply the thread to the side of the rule EF , so as to keep it always stretched as the rule is moved along, the point of this pin will describe the Parabola GHO .

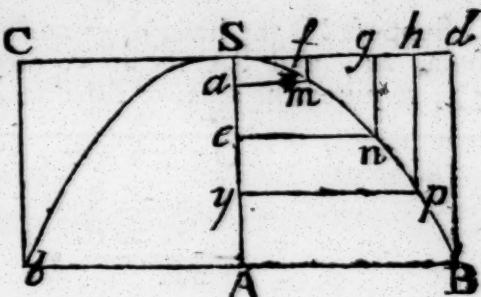
In the above figure, the right line AL is called the Directrix. The point C is the Focus of the Parabola. All perpendiculars to the Directrix, as LK , RQ , are called diameters; the points where these cut the Parabola, are called its Vertices; the diameter BI , which passes through the Focus C , is called the Axis of the Parabola, and its Vertex, H , the principal Vertex. A right line terminated on each side by the Parabola, and bisected by a diameter, is called an Ordinate Applique, or simply the ordinate to that diameter. A line, equal to four times the Segment of that diameter, intersected between the Directrix, and the Vertex is called the Latus Rectum, or Parameter of that diameter. A right line which touches the Parabola only in one point, and being produced on each side falls without it, is a Tangent to it in that point.

The

The area of every Parabola is equal to two thirds of its circumscribing Parallelogram.

DEMONSTRATION.

Let the figure, SAB, represent half the Parabola; draw dB , parallel to the Axis SA, and Sd parallel to the semi-ordinate AB; and suppose Sd to be divided into an infinite series of equidistant points, as $f, g, h, \&c.$ and from these points imagine a series of parallel lines, viz. $fm, gn, hp, \&c.$ to touch the curve of the Parabola, and meet the semi-ordinates, $ma, ne, yp.$



Then it will be $\begin{cases} \text{As } SA : AB^2 :: Sa : am^2. \\ \text{As } SA : AB^2 :: Se : en^2. \\ \text{As } SA : AB^2 :: Sy : yp^2, \&c. \end{cases}$

But $Sa = fm.$ $Se = gn.$ $Sy = hp.$ $SA = dB.$

Therefore alternately it will be $\begin{cases} AB^2 : dB :: yp^2 : hp. \\ AB^2 : dB :: en^2 : gn. \\ AB^2 : dB :: am^2 : fm. \end{cases}$

In these proportions $am^2, en^2, yp^2, \&c.$ are a series of squares, whose roots $Sf, Sg, Sh, \&c.$ are in arithmetical progression, beginning at the point S. And because the lines $hp, gn, fm, \&c.$ have the same Ratio; therefore they are a series of squares, wherein dB is the greatest term, and Sd , the Number of terms. Consequently $dB \times Sd =$ the sum of those lines. But

$SA \times AB = \overbrace{dB \times Sd}^3$: Therefore $SA \times AB =$ the sum of all the series of lines; but all those lines constitute the area of the semi-parabola's complement; or the area of what half the Parabola wants of completing, or filling up the Parallelogram $SdAB$.

Wherefore $\overline{SA \times AB} - \frac{1}{3} \overline{SA \times AB} = \underline{\underline{2SA \times AB}}$, will
 be the area of half the Parabola SAB. Consequently
 $\frac{2}{3} SA \times bB$, will be the area of the whole Parabola bSB.

EXAMPLE.

Suppose bB the base, or greatest ordinate of a Parabola, 24; and SA its intercepted diameter, or Axis 33; required the area, or superficial content?

OPERATION.

$$66 = 2 SA.$$

$$24 = bB.$$

$$\underline{\quad 264 \quad}$$

$$\underline{\quad 132 \quad}$$

$$1584 = 2SA \times bB.$$

Then 3) 1584 (528. The area required.

$$\underline{\quad 15 \quad}$$

$$\underline{\quad \cdot \cdot 8 \quad}$$

$$\underline{\quad 6 \quad}$$

$$\underline{\quad 24 \quad}$$

$$\underline{\quad 24 \quad}$$

B O O K II.

The application of the above Rules to the actual MENSURATION of all kinds of SUPERFICIES.

Philo. **W**E have now, my dear Discipulus, gone through the several rules, on which the art of practical Mensuration is founded; let us therefore now proceed to shew the methods used by artists in putting those rules in practice: and I will venture to say, that if you listen with the same attention you have hitherto done, you will find no difficulty in this part of the task.

Discipulus. You may depend upon it, Sir: nothing in my power shall be wanting. Indeed it would be very ungrateful, if I did not set the highest value on your instructions, as I have already profited by them in so remarkable a manner.

Philo. It gives me the highest pleasure to find you are so fond of your learning: I wish all youths would imitate your example; they would never then think their tasks irksome, nor the pains they took in understanding their lessons spent in vain. But leaving these reflections, we will proceed to shew the best methods of putting in practice the foregoing rules of Superficial Mensuration.

C H A P. I.

Of the art of MEASURING LAND; generally called SURVEYING of LAND.

SURVEYING, or the Art of Measuring Land, implies the act of taking the dimensions of any field, parcel, or tract of land, laying down the same in a map or draught, and casting up the contents, by the methods already delivered for the mensuration of superficies.

As this art is of the utmost importance to all owners and occupiers of land, I shall endeavour to explain it in the easiest and most intelligible manner; and shew how any field, of whatever figure it be, may be measured, protracted, and cast up, with the help of chains, compasses and scales: and afterwards endeavour, in the

most easy and concise manner, to shew the use of the plain table and other instruments used in surveying.

S E C T. I.

Of Surveying by the CHAIN, COMPASSES, and SCALES only.

This method of measuring land is generally used where the quantity is small, as that of one, two or three fields; and therefore often called for: as the farmer frequently agrees with reapers and mowers, to reap or mow his corn at a certain price, for each acre. But before we proceed to explain this method of surveying, it will be proper to describe the necessary instruments.

Concerning CHAINS, COMPASSES, and SCALES.

I. Amongst the many sorts of chains used for measuring land, three are chiefly esteemed, and which bear the names of their inventors, Rathbone, Gunter, and Wing; all of them ingeniously divided, and useful in their kind; but the description of one being sufficient for our purpose, we shall describe that invented by Mr. Gunter, as being most in use, and easy to be procured.

This chain contains in length four statute-poles or perches, each perch containing 16 feet and a half, or 5 yards and a half; so that the whole chain is 66 feet, or 22 yards long.

This chain is divided into 100 equal parts, or links, whereof 25 are a just pole or perch; and for ready counting, there is usually a remarkable distinction by some plate or large ring at the end of every 25 links, but especially at the middle of the chain, which should differ from the rest in greatness and conspicuousness. Also at the end of every tenth link, it is usual to hang a small curtain-ring; and if there be at every five links end a piece of wire, made like the bow of a link, with a small shank, an inch or less long, or some such distinction, it will be still better.

When you are to measure any line by this chain, you need regard no other denomination than chains and links, set down with a dot of your pen betwixt them.

For

For example, if you find the side of a close to be 6 chains and 35 links long, it is thus to be put down, 6. 35.

But if the links be under 10, a cypher must be prefixed; so 7 chains 9 links must be thus expressed 7. 09.

II. In the using of this, or indeed any other chain, care must be taken both to go strait, and to keep a true account; for which purpose it is necessary that the person who goes before should carry a bundle of rods, to stick down at one end of the chain, having first stretched it well; and that he who follows, should gather up the rods, keep the account exact, and also, at every remove, observe whether the leader is directly between his eye and the angle, or any other mark, towards which he is measuring.

III. Compasses are so well known, that it is needless to give any other description of them, except that they should be made of brass, with steel points, small and neatly wrought, 9 or 10 inches long, from the joint to the points, turning so truly upon the rivet, that they may be easily opened, and yet stand so firmly, that an arch or circle may be, without their shrinking, described upon a large Radius.

IV. Scales are certain lines divided into equal parts, upon plates or broad rules of brass or box, and they are of two sorts, *first*, plain; *second*, diagonal.

i. Plain scales are made up of two small lines, parallel to one another, and at a little distance, and these are divided into large equal parts, which signify tens, and are noted 10, 20, 30, 40, 50, &c. according to the length of the lines.

They may be of any convenient length; but these large divisions are seldom more than inches, or less than third parts of an inch.

Again one of the large divisions is sub divided into ten equal parts by short lines, whereof that in the middle, standing for five, is longer than the rest.

According to the numbers of these little parts contained in an inch, it is named, A Scale of 10, 11, 12, 16, 20, 24, 30, &c. in an inch. The annexed figure marked A, is of 10 in an inch, so noted at the top, as is usual upon the rules, and indices of plain tables. The line marked O Q, separates units and tens; units being taken upward from that line, and tens downward, but mixt numbers both ways.

EXAMPLE.

Seven is the extent of the compasses upon the scale A, from the line OQ to K; 30 is their extent from the line OQ to the bottom; and 27 is the extent from the line 20, to the short line K.

These plain scales, especially the smaller sorts of them, such as 24, or 30 in an inch, are very proper for drawing figures upon paper, where the numbers represented by the lines are not above 100, for then every division may be counted as it is upon the scale, or above upon a longer scale.

Also, in the surveying of forests, chases, and extensive commons, where the lines are vastly long, and the mistake of a few lines is not considerable, they may be conveniently used, accounting the tens and units, for so many whole chains, and so estimating the parts of a chain with the compasses, upon the small divisions. But it would be much better, for ordinary measuring, if the grand divisions of the scale were each two inches, for then the smaller divisions being of 5 in an inch, would be so large as to be subdivided into 5 each, which represents 20 links; and then the half of one of those lesser divisions, signifying 10 links, and the quarter 5, it is rendered easy for the most common capacity, to come very near the truth by estimation.

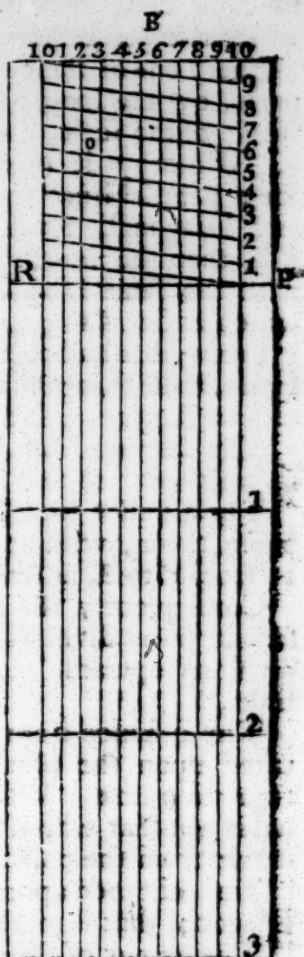


2. The Diagonal Scale is so well known to every mathematical instrument-maker, so easy to be procured, and every way so fitted to Gunter's chain, that we cannot but highly recommend it to every surveyor.

It is made as appears by the figure in the margin, upon 11 parallel equi-distant lines, so as to include 10 equal spaces, which are all cut at right angles by transverse lines, dividing them all into 4 equal parts.

One of these transverse lines, viz. PR, where it toucheth the first and last lines, separates between the hundreds, or whole chains, and the tens, representing 10 links a piece, the chains being numbered downwards on the left hand from P only to 3, but on the instrument itself they may go on to 9 or 10; but the tens upwards from 1 to 10.

From the points of division into tens upon the first line, beginning at P, to the like points beginning at R in the last line, are 9 Diagonal lines drawn, the first beginning at P, and ending at the first division above R. The second beginning at the first division above P, and ending at the second above R. In a word, they are all drawn from one division less from P, to one more from R; by which means every Diagonal, by that time it hath passed from the first line to the eleventh, is a whole tenth part of an inch, which answers to ten links of the chain, farther distant from



the line P R, than at the points upon the first line whence it was drawn.

Every one of these Diagonals is divided into ten equal parts, by the long parallel lines running through the whole of the scale, and numbered on the top from 1 to 9. Whereby it is evident, that the intersection of the 9 parallel lines that are numbered at the head with any Diagonal, must be farther distant from the line P R, than the intersection of the line next before it, with the same Diagonal by $\frac{1}{10}$ of $\frac{1}{10}$; that is, by $\frac{1}{100}$, which answers to a single link of your chain.

From what hath been said, and the inspection of the figure, these things plainly follow: which, as so many clear instances, will help you to understand it fully.

The distance from P R, to the second division below it, answereth to two chains.

The distance from P R, to the eighth division upward, being taken, with compasses, upon the first line of the 11 from P to 8, answereth to 80 links.

Consequently the extent of the compasses from the second grand division below P, to the eighth of the less divisions upward, is proportionable to two chains 80 links.

The distance from P R, to the first Diagonal being taken upon the parallel line, noted with 9 above, answers to 9 links: Where observe, that the first Diagonal is not that which is noted with 1, but that which is drawn from the point P.

The distance upon the same line, from P R to the Diagonal that is marked with 7, is equal to 79 links.

The extent of the compasses from the bottom of the figure upon the same line to the same Diagonal answers to 3 chains 79 links.

In short, the chains may be measured upon any line P R, to the given division noted with the given number Decads alone, or chains and Decads upon the first line where the Diagonals begin. Links alone, Decads with links and chains, and Decads with links always upon that line, where the number of the odd links stand at the head of the scale.

How to cast up the content of a Figure, the lines being given in chains and links.

Having described these plain instruments, and in some measure shewed the use of them in general, it is very proper in the next place, to teach their joint use in measuring and protracting; but because I would have you and every young surveyor, before I take him into the field, able to perform the work entire, I intend to shew:

1. How he ought to make his computations. 2. The grounds, or principles that will justify him in so doing.

For the first, the following rules are necessary.

1. Put down your length and breadth of squares and oblongs, and your bases and half perpendiculars of triangles, directly under one another, expressed by chains and links, with a dot betwixt, as we have before observed.

2. If the odd links are under ten, put a cypher before the numeral figure expressing them; and if there be no odd links, but all even chains, put two cyphers after the dot.

3. Multiply length by breadth, and base by the half perpendicular, according to the rules already given for finding the content of figures.

4. From their product cut off five figures, accounting cyphers also for such, reckoned from the right hand backward, with a dash of your pen, so shall these to the left hand signify acres.

5. If those 5 figures cut off be not all cyphers, multiply them by 4, and cutting off 5 towards the right hand again, the rest will be roods, or quarters.

6. If amongst these 5 figures towards the right hand, that were cut off at the second multiplication, there be any figures besides cyphers, multiply all the 5 by forty, and cutting off 5 again by a dash of your pen, those on the left hand signify square perches, poles, or roods.

This will be more fully explained by the following examples.

THE WHOLE ART

QUESTION I.

What is the content of a square whose sides are every one of them 7 chains, 25 links?

$$\begin{array}{r}
 \text{Length } 7. \quad 25. \\
 \text{Breadth } 7. \quad 25. \\
 \hline
 3625 \\
 1450 \\
 \hline
 5075 \\
 5 \overline{) 25625} \\
 \underline{40} \\
 102500 \\
 \underline{40} \\
 100000
 \end{array}$$

Ans. Five acres, 1 Rood, and 1 Perch, as appears by the above operation.

QUESTION II.

In a long square whose length is 14 chains, and the breadth 6 chains 5 links, what is contained?

$$\begin{array}{r}
 \text{Length } 14.00 \\
 \text{Breadth } 6.05 \\
 \hline
 7000 \\
 8400 \\
 \hline
 847000 \\
 \underline{4} \\
 188000 \\
 \underline{40} \\
 3520008
 \end{array}$$

Ans. Eight acres, 1 rood, and 35 perches, as the Work makes it evident.

QUESTION

QUESTION III.

In a triangle, whose base is 3 chains, and half the perpendicular 98 links, what is the content?

The base 3.00

Half perpend. 0.98

$$\begin{array}{r}
 2400 \\
 2700 \\
 \hline
 29400 \\
 4 \\
 \hline
 117600 \\
 40 \\
 \hline
 7104000
 \end{array}$$

ANSW. No acres, 1 rood, 7 perches.

There are other ways of computation by scales, tables, &c. as will appear by the following observations.

1. It is evident, that by this method of multiplication, the product is square links; for every chain being 100 links, the multiplying 7.25 by 7.25, or 725 by 725 is the same with or without dots; for the dots signify something as to conception, but nothing in operation. The product therefore of the first example was really 525625 links.

2. Every chain being 4 perches long, it follows, that 5 chains, or 20 perches in length, and 2 chains, or 8 perches in breadth make an acre, or an 160 square perches; for 20 being multiplied by 8, gives 160.

3. From hence it plainly follows, that there are exactly 100000 square links in an acre; for 5 chains multiplied by 2, is the same with 500 links by 200, which makes 100000. And it is an old but plain rule, that when the divisor consists of 1 and cyphers, as 10, 100, 1000, 10000, 100000, &c. cut off from the right hand as many figures of the dividend as the divisor hath cyphers, accounting them the remainder; so will the rest on the left side be the quotient. It is evident then, that 525625 square links, make 5 acres, and 25625 square links over.

Thus, as far as relates to acres, we have made it plain to every capacity, that the rules for computation

are good. We shall next proceed to roods and perches, though it might be sufficient to mention that known rule in decimal arithmetic, which multiplying decimal fractions by known parts, gives those known parts in integers, due regard being had to the separation. We shall proceed thus : if 25625 square links, which remain above an acre, contain any quarter or quarters of an acre, then if they be multiplied by 4, and divided by 100000, that is 5 cut off from the product, they will contain so many acres as now they do quarters, or roods, for any number of quarters multiplied by 4, will of course produce the like number of units or integers ; the division only reduces them into the right denomination. Now, 25625 being multiplied by 4, and 5 figures being cut off from the product, the result is, 102500, that is an acre and above ; which shews it was above a quarter before it was multiplied by 4.

And to find how much, that is, how many square perches are contained in this last remainder, you must consider this 2500, not as square links remaining above the rood or quarter, but as 4 parts or quarters of square links, or which is the same, as the true number of square links, multiplied by 4, and consequently being multiplied by 40, the fourth part of the square perches in an acre, it must as often contain 100000 square links, or an acre, as the quarter of this number 2500, viz. 625 signifying square links, contains square perches ; for 100000 divided by 160, the number of perches in an acre, gives 6.25, answering to 1 perch, and 2500 multiplied by 40, gives 100000, or 1 acre, the 5 cyphers being cut off.

2500	160	100 000625
<u>40</u>		<u>960</u>
1,00000		400
		<u>320</u>
		800
		<u>800</u>

How

How to measure a close, or parcel of land, and to protract it, and give up the content.

All closes, or parcels of land, are either such as need not be plotted for finding their true measure, but the chain alone does the work; or such as cannot be conveniently measured without plotting or protraction.

Of the first sort are the square and long square, known before hand to be such, or found so to be by such instruments as are not yet described, or by measuring all the sides and Diagonals. These squares and Parallelograms need no protracting; for you need only multiply the chains and links of the breadth, and proceed as in the first and second examples; but all other figures whether triangular or multangular, are to be protracted. We shall give examples therefore in the 3 sorts of figures, Triangular, Quadrangular, and Multangular.

But before we proceed to particular instances, let the young practitioner remember,

1. To begin at some particular angle of the field, where there is a house, gate, stile, well, or other mark; and if there be none, then to dig up a clod, and drive a stake, or at least observe, whether it is situated in the east, west, north, or south corner of the field, and on your paper mark it with the letter A, or any other.

2. To go parallel to the side of the field, if you are not obstructed by pits, bushes, &c. and if you are, to allow for it; accustoming yourself to go constantly with the same hand towards the hedges, walls, or pales; and when you go contrary to your usual custom, note it on your paper, by some mark known to yourself.

3. To set down the chains and links of every side as you measure, and not trust your memory; for which purpose, a black lead pen will be very proper.

4. To take care if you have more scales than one upon your rule, that you do not confound yourself by taking off lines from several scales, or measuring perpendiculars upon wrong ones; for every line of the same figure must be made by the same scale, and the perpendiculars measured by it.

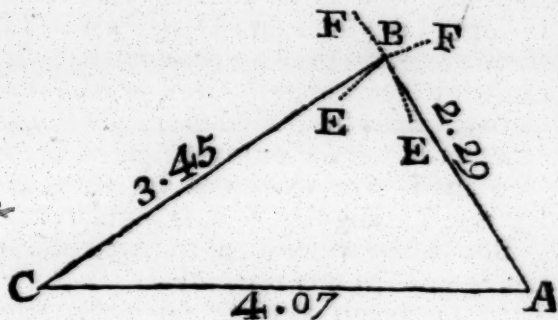
5. To make use of a scale of larger divisions when you measure small closes, and of smaller when you measure great ones.

6. To make your lines and points where angles meet, small, pure, and neat.

7. To set on your chains and links at twice, when any line is too long for your scale.

These things being premised, proceed thus :

I. Suppose a triangular field is to be measured with the chain : begin at the eastern angle A, and find the sides in their order and measure to be severally thus ; 2.29, 3.45, 4.07.



Making use of the less Diagonal scale, because the other would make the figure too large, otherwise it were more proper for so small a close ; with your compasses take off the scale 4 chains and 7 links, and setting them from A to C, draw that line for the base, as being the longest of the three ; then take 2 chains 29 links off the same scale, and set them in the eastern point A, and turning the loose foot of the compasses above the line at C, describe at that distance 2.29, the arch E F.

Next taking with your compasses upon the same scale, the extent of 3 chains 45 links, place one foot in the point C, and with the other make the arch F E, intersecting the former in the point B ; and drawing the lines A B, and B C, the triangle A B C is the plot of the triangular field measured.

But before you can give up the content, you must find the length of the perpendicular, which is done by setting one foot of the compasses in B, and extending the other to the base A C, so as to touch, without passing over it, for then the length of the perpendicular is between the points of the compasses, and being applied

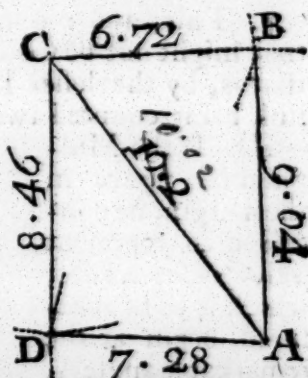
to

the points of the same scale, by which the triangle ABC was made, it appears to be 1 chain 42 links. With the half whereof multiply 4.07, the length of the base, the content appears to be no acres, 1 rood, 6 perches.

$$\begin{array}{r}
 \text{The base} \quad 4.07 \\
 \text{Half perpend. } 0.71 \\
 \hline
 407 \\
 2849 \\
 \hline
 28897 \\
 \hline
 4 \\
 115588 \\
 \hline
 40 \\
 \hline
 6123520
 \end{array}$$

II. Suppose you were to measure a quadrangular, or four-cornered field, begin, as before, at some remarkable angle, and going round the close find the sides to be 9.04, 6.72, 8.46, 7.28, and the Diagonal from that remarkable angle to the opposite angle to be 10.02; we begin to protract it thus:

Having by the help of your scale and compasses drawn the Diagonal 10.02, from the remarkable angle A to C, the opposite angle, make a triangle of it, and the first and second sides 9.04 and 6.72, and another after the same method of that Diagonal, and the third and fourth sides 8.46 and 7.28, so you have the Trapezium ABCD.



Then by the help of your scale and compasses, you will find the perpendicular of the triangle ABC to be 6.02, and of the other, viz. CDA 6.01, which added, are 12.03, whereof the half sum is 6.01; by which multiplying the base 10.02, the content of the field will be 6 acres, no roods, 3 perches.

$$\begin{array}{r}
 \text{The base} \quad 10.02 \\
 \text{The perpend.} \quad 6.01 \\
 \hline
 1002 \\
 60120 \\
 \hline
 6|02202 \\
 \hline
 4 \\
 108808 \\
 \hline
 40 \\
 \hline
 3152320
 \end{array}$$

Before we proceed any farther, it is necessary to observe,

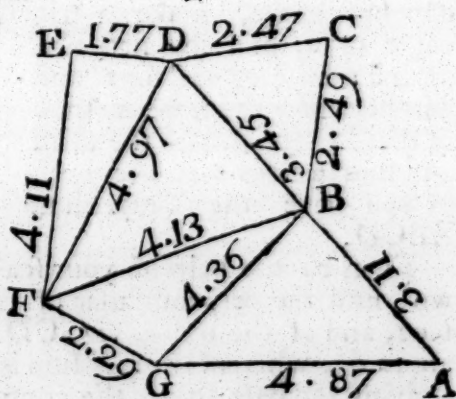
1. Any Quadrangular close, or parcel of ground whatsoever, having right lines, may be thus measured, protracted, and computed.

2. The odd measure below perches is not regarded here, nor in the former computations, being always under a square perch; but in Multangulars, where there are many remainders, they must be summed up, and the perches contained in them added to the content before found.

3. This, and the following figures, are made, that they might not be too large, by a scale of 400 in an inch, that is, by the lesser Diagonal scale, each chain and link being counted two.

III. If the Multangular figure in the margin, be conceived to represent a close of seven sides, which is to be measured, begin at the remarkable angle A, and going round the close, find the sides, will be 3.11, 2.49, 2.47, 1.77, 4.11, 2.29, 4.87.

Then measure the four Diagonals, BD, DF, FB, and BG, in order, you will find them to be 3.45, 4.97,



4.97, 4.13, and 4.36; which is as short a way as can be taken to prevent unnecessary walks.

But when you come to protract by the help of your scale and compasses, first make the triangle BCD of the first diagonal, and the second and third sides. Then the triangle DEF upon the second Diagonal, and fourth and fifth sides; and upon the same Diagonal, as a common base, the triangle BDF of the first, second and third Diagonals.

Next of the same third Diagonal, together with the fourth Diagonal and the sixth side, make the triangle BFG, and upon the fourth Diagonal, as upon a common base with the first and last sides, the triangle ABG. So is the whole close plotted.

And now it stands visibly reduced into two Trapezias ABFG and BDEF, together with the triangle BCD, which we shall not now cast up, having already frequently shewn the method of doing it.

But it must be acknowledged that this method of plotting parcels of land, which have many angles, requires not only more care and pains, but better skill and memory, than to draw Diagonals upon paper, when the plot is already taken by the plain table, or other standing instrument. We shall therefore to assist our young practitioner, inform him of two ways to prevent his being liable to any mistake.

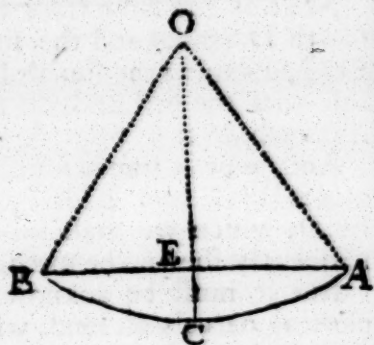
One is to divide the Multangular field into two or more parts, as the last might have been by the Diagonal BF; then might each part have been measured severally, as if they had been separated by a pale, or were the property of different men, parted by a boundary.

The other is to draw a rude draught of the figure of the land you intend to measure, not only as to the sides, but also necessary Diagonals. Then measuring the lines upon the ground, correspondent to those on the paper, set the lines as you measure them upon the lines of the draught, as if it were the true one, and when you have finished the measuring, protract it truly.

Concerning the measuring of circles, and their parts.

It is very rare, that a Land-Meter ever has occasion to measure any field, or parcel of land, that will prove either Circle, Semi-circle, Quadrant or Sector. Sometimes, indeed, there will be a little crook in an old hedge bowing like an arch; tho' I have never seen any offer to measure it as a Segment, but always take it as an angle, or angles.

But that which most frequently occurs in Mensuration, though seldom much regarded, except where a minute exactness is required, is that particular sort of Segment, which is less than a semi-circle, such as this figure ABC. And to find the content of it, the center of the circle, whereof this is a Segment, must be first found, as here at O, from which lines drawn to A and B, make up the Sector AOB; which being measured according to the last rule, and from the content thereof, the content of the triangle AOB subtracted, the difference or residue is the content of the Segment ABC.



But two questions may be here demanded.

1. How may the center be found? 2. How may such a portion of land be truly protracted and computed?

To the first we answer, That the most exact and artificial way is, by making a mark any where in the arch,

As for EXAMPLE,

At the point C, and then, by a Problem known not only to every Surveyor, but to ordinary Carpenters and Joiners, for finding the center of a circle, whose circumference will pass through three given points that are not in a right line, as ACB, to find the center O. But if you know not how to do it, cross the line AB in the middle as is here done by the perpendicular OC;

OC; so you may by a few trials find both the due extent of your compasses, and the point in your perpendicular, that will fit your purpose near enough; for if a small error be committed in making up the Sector, the most of it goes off again in the subtraction of the triangle.

II. For the latter you may take this ready course; measure the length of both your lines, the chord and the arch, and their distance at the middle of them both; then when you come to protract, first take the length of your right line from the scale, and having laid it down, cross it in the middle at right angles with a dry line, as in the last figure, so shall it intersect the line AB in the point E; then from the same scale, take the measured distance between the 2 lines in the middle, and set it off upon that dry line from the intersection E, to the point C. Then by trials, find a due place in the dry line OEC, and at such a distance with your compasses, that the one foot resting in that line, the other may describe the arch ACB, and the Sector is protracted.

Concerning customary measure, and how it may be reduced to statute measure, et e contra, either by the rule of three, or a more compendious way, by multiplication only.

Though the statute-perch, or pole, is 16 feet and a half, and no more, yet there are poles of larger measure used in many places, as of 18, 20, 21, 24, and 28 feet, in some 22 and a half. It were therefore very convenient, that our young surveyor were furnished with a chain, fitted to the customary measure of the country where he lives. But because these are too large and cumbersome for small closes, it is very convenient, instead of one chain of 100 links, to make two of 2 poles a piece, each pole divided into 25 links, as that of 100 is; which two half chains may, in the measuring large fields be tied together by the loops with pack-thread, or joined by a buttoning key-ring for more speedy dispatch; but in smaller we may use the half chain of

50 links, only taking care, that we count not half chains for whole ones.

And in these cases where the poles are large, and the closes small, it were still more convenient if you had a chain of two poles only, divided into 100 links. But be sure to remember, when five figures are cut off in the first multiplication, that the content is only roods, and parts of a rood.

We have already observed, that the length of the pole is different in different parts of the kingdom; but there is no occasion for having chains with various sorts of links; since it is very easy to reduce statute measure into customary; and consequently, Gunter's chain will be sufficient in every part of England.

In order to this, it must be remembered, that acres are in proportion to one another, as the square of their poles; and therefore, if you multiply 33, the number of half feet in the statute-pole, by itself, which gives 1089, and also multiply the number of half feet contained in a pole of that measure you would reduce into; in the same manner you may, by the rule of three reverse, obtain your desire, making for that purpose 1089, the first number, the statute measure the second, and the squared half feet of the pole given the third. As for example: Suppose of a close measured by the statute pole, the length and breadth, and their product as are represented in the margin. And

it is desired that the content may be cast up according to the large Cheshire measure of 8 yards, or 24 feet to the pole or rood. Then before you cut off any figures, consider that in the statute pole are 33 half feet, and the Cheshire pole 48;

multiply therefore 33 by 33, and 48 by 48, and you will have 1089 and 2304, which, together with the said product, may be thus placed; 2304 : 672693 :: 1089, and so multiplying 672693 by 1089, and dividing their product, being 732562677, by 2304, the quotient is 317952; from which, if five figures towards the right hand be cut off, the content, by this customary measure

$$\begin{array}{r}
 9.33 \text{ L.} \\
 7.21 \text{ B.} \\
 \hline
 933 \\
 1866 \\
 6531 \\
 \hline
 672693 \text{ Product.}
 \end{array}$$

sure of 24 feet to the pole, will be 3 acres, no roods, 28 perches, as appears below :

$$\begin{array}{r}
 3117952 \\
 \hline
 4 \\
 171808 \\
 \hline
 40 \\
 28172320
 \end{array}$$

But if the lines on the land had been measured according to that custom of 24 feet to the pole, and the content must have been found according to the statute measure, then we must have multiplied the product by 2304, and have divided that latter product by 1089. And in the same method you may proceed in all, or any of the rest. But though this way is very exact, plain, and comprehensive, suiting all the customary measures before mentioned, without fractions; it is certainly somewhat tedious, except the practitioner knows how to relieve himself, by a large table of Logarithms, which we cannot here treat of. Therefore, to contract the work in some degree, take notice, that all the customary poles above-mentioned, except those of 20 and 28 feet, which are rarely used, they are each capable of being divided into half yards: And therefore, if instead of squaring the half feet, you square the half yards of both poles, and work with them, you will attain the same end without any material difference, the small diversity that there is, being generally in the useless remainders, not in the least affecting the desired quotient, that gives the answer near enough for use.

As for EXAMPLE.

If we had squared 11, the number of half yards in the statute pole, which would make 121, and also 16, the number of half yards in the Cheshire pole, which would make 256, and then multiplied the first product 672693 by 121, the second product would have been 81819853, which being divided by 256, the quotient would have been, as before, 317952. And this way is, in a manner, co-incident with Mr. Holwell's first method.

11
11
11
11
121
16
16
96
16
256

Take notice also, that whether you use either of these, or the following methods, you need not reduce the particular squares, triangles, or trapezia's severally; but sum up all their products together, and then reduce all at once.

But if you would reduce statute measure into customary, by multiplication, only observe the following table:

The Content	{ .84027 }	Gives the	{ 18 }	Feet.
by the sta-	{ .68062 }	content	{ 20 }	
tute pole,	{ .61734 }	by the	{ 21 }	
being mul-	{ .53777 }	pole of	{ 22½ }	
tiplied by	{ .47265 }		{ 24 }	
	{ .34725 }		{ 28 }	

The Use of this TABLE.

When you have multiplied lengths by breadths, or bases by half perpendiculars, multiply these products by the decimal fractions, answering to the customary measure, into which you would reduce statute measure, and from that latter product, first cut off five places towards the right hand, as not to be regarded, being only parts of a square link, then cutting off 5 more, and proceeding to multiply by 4, and then by 40, you will have the content by that customary measure.

EXAMPLE.

EXAMPLE.

Suppose the length of a close, measured by Gunter's chain and multiplied by the breadth, measured also by the same, produced 672693 square links; and it is desired, that the content may be given in Cheshire measure of 24 feet to the pole; you must multiply 672693 by 47265, the decimal fractions answering to 24 feet, and from that product, being 31794834645, cut off and cast away 5 places, and the rest being 317948, are in the usual way easily reducible into 3 acres, no roods, and 28 perches, which is what it amounted to in the former method exactly.

$$\begin{array}{r}
 317948 \\
 \times 47265 \\
 \hline
 1589740 \\
 25116870 \\
 214361400 \\
 1589740000 \\
 \hline
 31794834645
 \end{array}$$

But if you measured by a chain of customary poles, and desire to know what the content is in statute measure, the following table must be used :

The content measured by the pole of	$\left\{ \begin{array}{l} 18 \\ 20 \\ 21 \\ 22\frac{1}{2} \\ 24 \\ 28 \end{array} \right\}$	Being	$\left\{ \begin{array}{l} 1.19008 \\ 1.46922 \\ 1.61983 \\ 1.85950 \\ 2.11570 \\ 2.87970 \end{array} \right\}$	Gives the content by the statute pole.

To understand which, observe this EXAMPLE.

Suppose the length and breadth of a long square, being measured by a chain of 24 feet to the pole, and multiplied together, make their product 317952, let this be multiplied by 2.11570, which answereth to 24 feet, and the latter product will be 67269104640; from which, if you cut off, and cast away 5 places towards the right hand, the remainder is 672691; which, in the usual way, is easily reduced to 6 acres, 2 roods, and 36 perches.

$$\begin{array}{r}
 317952 \\
 \times 2.11570 \\
 \hline
 2195664 \\
 6359040 \\
 67269104640 \\
 \hline
 67269104640
 \end{array}$$

If the content to be reduced, be cast up into acres, roods, and perches, reduce all into perches, and then in other respects work as before, either by the rule of three, or by this last method of multiplication only.

So

So shall you have the content in square perches, according to the measure desired, which you may reduce into acres by dividing them by 160; and if any thing remain, that remainder being divided by 40, will give the roods in the quotient, and the latter remainder the number of square perches.

For trial of which rules, observe the answer of these two following questions, wrought all three ways.

QUEST. I. How many acres, roods, and perches, according to the pole of 11 feet, are contained in 5 acres, 3 roods, and 11 perches, statute measure?

ANSW. Four acres, 3 roods, and 22 perches, as appears by the following work:

FIRST METHOD.
A. R. P.

33	36	5	3	11
<u>33</u>	<u>36</u>	<u>4</u>		
99	216	23		
<u>99</u>	<u>108</u>	<u>40</u>		
1089	1296	931		
1089	931	:: 1296		
	<u>1089</u>			
	8379			
	7448			
	<u>9370</u>			
	1013859			
1296)	1013859	(782		
	<u>9072</u>			
	10665			
	<u>10368</u>			
	2979			
	<u>2592</u>			
	387			

160)	982	(4
	640	
40)	142	(3
	<u>120</u>	
	22	

SECOND

SECOND METHOD.

$$\begin{array}{r}
 11 \quad 12 \\
 \hline
 11 \quad 12 \\
 \hline
 11 \quad 24 \\
 \hline
 11 \quad 12 \\
 \hline
 121 \quad 144
 \end{array}
 \qquad
 \begin{array}{r}
 144 : 931 :: 121 : 782 \\
 \hline
 121 \\
 \hline
 931 \\
 1862 \\
 \hline
 931 \\
 \hline
 112651 \quad (782) \\
 1008 \\
 \hline
 1185 \\
 1152 \\
 \hline
 331 \\
 288 \\
 \hline
 43
 \end{array}$$

$$\begin{array}{r}
 160) \quad 782 \quad (4) \\
 \hline
 640 \\
 40) \quad 142 \quad (3) \\
 \hline
 120 \\
 \hline
 22
 \end{array}
 \left. \vphantom{\begin{array}{r} 160) \quad 782 \quad (4) \\ 40) \quad 142 \quad (3) \end{array}} \right\} \begin{array}{c} \text{Or} \\ \text{thus} \end{array} \left\{ \begin{array}{r}
 40) \quad 782 \quad (19) \\
 \hline
 40 \\
 382 \quad 4) \quad 19 \quad (4) \\
 \hline
 360 \\
 \hline
 22 \quad 3
 \end{array}
 \right.$$

THIRD METHOD.

$$\begin{array}{r}
 84027 \\
 \hline
 931 \\
 \hline
 84027 \\
 252081 \\
 \hline
 756243 \\
 \hline
 782.29137
 \end{array}$$

$$\begin{array}{r}
 160) \quad 782 \quad (4) \\
 \hline
 640 \\
 40) \quad 142 \quad (3) \\
 \hline
 120 \\
 \hline
 22
 \end{array}$$

QUEST. II. How many acres, roods, and perches of statute measure, are contained in 8 acres, 3 roods, or quarters, and 21 perches of 21 feet to the pole?

ANSW. Fourteen acres, 1 rood, and 21 perches, as appears by the three following works in the several methods.

FIRST METHOD.

A. R. P.

8 : 3 : 21

$$\begin{array}{r}
 4 \\
 \hline
 35 \\
 40 \\
 \hline
 1421
 \end{array}$$

$$\begin{array}{r}
 42 \quad 33 \\
 \hline
 42 \quad 33 \\
 \hline
 84 \quad 99 \\
 168 \quad 99 \\
 \hline
 1764 \quad 1089
 \end{array}$$

$$1089 : 1421 :: 1764 : 2301$$

$$\begin{array}{r}
 1764 \\
 \hline
 5684 \\
 8526 \\
 9947 \\
 1421 \\
 \hline
 1089) \quad 2506644 \quad (2301 \\
 \underline{2178} \\
 3286 \\
 \underline{3267} \\
 1944 \\
 \underline{1089} \\
 855 \\
 160) \quad 2301 \quad (14 \\
 \underline{160} \\
 701 \\
 \underline{640} \\
 40) \quad 61 \quad (1 \\
 \underline{40} \\
 21
 \end{array}$$

SECOND

OF MEASURING.

71

SECOND METHOD.

$$\begin{array}{r}
 14 \quad 11 \quad 121 : 1421 : : 196 : 2301 \\
 \hline
 14 \quad 11 \\
 56 \quad 11 \\
 \hline
 14 \quad 11 \\
 196 \quad 121
 \end{array}$$

$$\begin{array}{r}
 196 \\
 \hline
 8526 \\
 12789 \\
 \hline
 1421 \\
 121) 278516 \quad (2301 \\
 \hline
 242 \\
 \hline
 365 \\
 \hline
 363 \\
 \hline
 216 \\
 121 \\
 \hline
 160) 2301 \quad (14 \\
 \hline
 160 \\
 \hline
 701 \\
 \hline
 640 \\
 \hline
 61 \\
 40) 61 \quad (1 \\
 \hline
 40 \\
 \hline
 21
 \end{array}$$

THIRD METHOD.

$$\begin{array}{r}
 1. 61983 \\
 \hline
 1421 \\
 \hline
 161983 \\
 323966 \\
 647932 \\
 \hline
 161983 \\
 \hline
 2301.77843
 \end{array}$$

$$\begin{array}{r}
 160) 2301 \quad (14 \\
 \hline
 160 \\
 \hline
 701 \\
 \hline
 640 \\
 \hline
 40) 61 \quad (1 \\
 \hline
 40 \\
 \hline
 21
 \end{array}$$

How to measure a piece of land, with any chain, however divided; or even with a cord, or cart-rope.

If you can procure a chain, and find it is not divided in the manner we have already explained, but into feet, or quarters of yards, or any such vulgar divisions, make no reckoning of the divisions, but measure it as exactly

as you can, to find out the true length of the whole chain; and if it fit none of those lengths before-mentioned, nor any of their halves, make it fit by taking off a link or two, or piecing it out with a string; then dividing the length of that chain by 100, or the half of it by 50, find the true length of a link, according to our artificial divisions, and having got a long stick, or rod, set as many of those link-lengths upon it as it will hold; then may you measure all the whole chains by your regulated chain, and the odd links of every line, by your divided stick, or rod, as will be proved by the following example.

Being far from any instrument, and requested by a friend to measure him a close, and you can procure a pair of compasses, an ordinary carpenter's rule of two foot, divided into inches and quarters, and also a piece of an old chain, seemingly divided into feet, you must measure it by the rule, and finding it to be 45 feet long, and some odd measure, piece it out with a strong cord that will not stretch, to 48 feet exactly; then it will serve for half a chain of 24 feet to the pole: This 48 multiply by 12, the number of inches in a foot, and that product being 576, divide it by 50, the number of links in half a decimal chain, and the quotient is $11\frac{66}{100}$ inches, or 11 inches and a half, and something over. Dividing therefore a long stick throughout into such parts, each containing 11 inches and a half, besides the breadth of the nicks, you are provided with tools to measure lines to a link with sufficient exactness.

In like manner you may proceed with a cord or rope, having fitted them to some known length; and then for protracting, it is easy with the compasses to make a plain scale of a large sort, either upon paper, or an even piece of wood.

Besides, there is a way of measuring the perpendiculars of triangles and trapezias upon the ground itself, so as to prevent the necessity of a scale; for if you have a small square with a hole in it, to turn upon the head of a stick, which you may fix where you please, as you are measuring the base of a triangle, or the diagonal of a trapezium; you may by a very few trials find
the

the place where the one leg will be just in the line you are measuring, and the other point at the angle from which the perpendicular falls on it, and then the space between your stick and that angle truly measured, is the perpendicular.

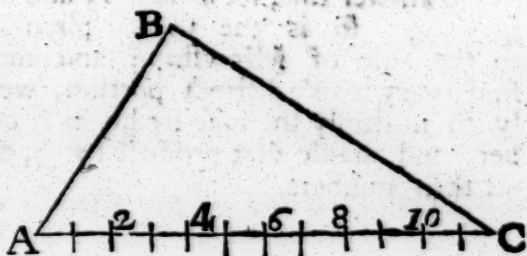
If you have not such a square, a square trencher, or any end of a board that hath one right angle, and two true sides, will supply the want of it. This is undoubtedly an excellent way of measuring a trapezium, though it be protracted afterwards; for by measuring the perpendiculars as aforesaid, and observing at how many chains and links end, the said perpendiculars meet the common base, the whole trapezium may be truly protracted without going about it; this little square sufficiently supplying the place of an instrument, which is usually called a cross or square, made up, as it were, of two small indices, like those for a plain table, but much less, with fore-sights and back-sights, and cutting one another at right angles put together, and having an hole at the center.

Concerning dividing of land artificially and mechanically.

I. To divide a triangle into any parts required; divide the base as the demand imports, then shall lines drawn from the points of division to the opposite angle, finish the division of the triangle.

EXAMPLE.

AC, the base of the triangle ABC, being divided into 12 equal parts, a line drawn from the angular point B to the point 6, divides the triangle into two equal parts; two lines drawn to 4 and 8, divide it into three equal parts; 2 lines drawn



G 3

to

to the points marked with 3, 6, and 9, divide it into four equal parts; and so lines drawn to 2, 4, 6, 8, 10, divide it into six equal parts.

It is likewise very obvious, that if the same triangle were to be so divided, that the one part should be double to the other, a line drawn from B to 4 or 8, doth the work, or if it be required to divide it into two parts, so as the one shall be triple to the other, a line drawn from B to 3 or 9, compleats the work. In like manner a line from B to 2 or 10 divides it into two parts, whereof the one is quintuple, or five-fold, to the other, and a line from B to 1 or 11, divides it into two parts, whereof the one is 11 times as large as the other.

Farther yet, if it were required that this triangle should be so divided, that the two parts should in quantity bear proportion, as 5 and 7, a line from B to 5 or 7, will answer the purpose.

The divisions will, indeed, be sometimes a little more intricate than this, yet not such, but that the seeming difficulty may be easily overcome, by observing the following method.

Suppose, a large triangle of common land to be divided amongst three tenants, A, B, and C, according to the quantity of their tenements: A, having 19 acres of land to his tenement, B 13, and C 7, the base of the triangle being found by measure to be 17 chains 27 links: and the demand is, Where the point of the division must be placed in the base, so as lines drawn from thence to the opposite angle, shall truly limit each man's part?

To answer this, let us add 13 and 07 to 19, and they give 39: so is the work plainly reduced to the rule of fellowship; and therefore, to find every man's distinct portion, we need only to multiply the base by his particular number, and divide that product by 39, the sum of all their numbers.

19

13

07

39

A.

$$39 : 17.27 :: 19$$

$$\begin{array}{r} 19 \\ \hline 15543 \\ 1727 \\ \hline 39 \overline{) 328.13} \quad (8.41\frac{1}{3}) \\ 312 \\ \hline 161 \\ 156 \\ \hline 53 \\ 39 \\ \hline 14 \end{array}$$

B.

$$39 : 17.27 :: 13$$

$$\begin{array}{r} 13 \\ \hline 5181 \\ 1727 \\ \hline 39 \overline{) 224.51} \quad (5.75\frac{1}{3}) \\ 195 \\ \hline 295 \\ 273 \\ \hline 221 \\ 195 \\ \hline 26 \end{array}$$

C.

$$39 : 17.27 :: 7$$

$$\begin{array}{r} 7 \\ \hline 39 \overline{) 120.89} \quad (3.09\frac{1}{3}) \\ 117 \\ \hline 389 \\ 351 \\ \hline 38 \end{array}$$

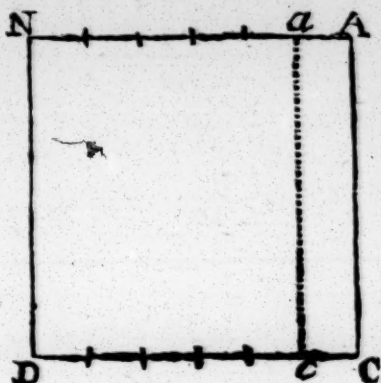
From these operations it is plain, that if we set off from the angular point where the base begins, 8 chains 41 links, and a little above the third part of a link upon the base for A, and where that ends, 5 chains and 75 links and $\frac{1}{3}$ of a link for B, and consequently, leave between this second division, and the other end of the base, 3 chains and almost 10 links, for C; lines drawn from those points of division to the opposite angle, will give each man his due.

What has been said, respecting the division of triangles upon their bases, will, with a little variation, serve for the dividing of all sorts of Parallelograms, either squares, long squares, Rhombus's, or Rhomboides; all the difference is, that instead of drawing lines from points in the base to the opposite angle, you must draw parallel lines from the points in one opposite side to another, as will be sufficiently plain by the following instance.

G. 4.

Suppose

Suppose the square figure to represent a close of six acres, and you are to cut off an acre at the side AC; having set off the 6th part of the line CD, from C towards D, as at e ; also the sixth part of the line AN from A towards N as at a : draw the line ea parallel to AC, and this takes off exactly a 6th part of an acre; that is, the small Parallelogram A C ea , is one acre.



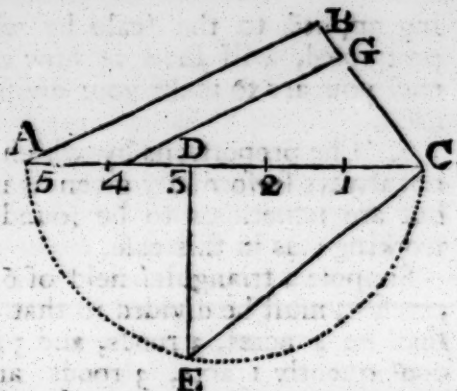
If it be not thought convenient, as in some cases it is not, to cut off a piece so long and narrow, you may, by the rule of three, find what other length of any greater breadth, will contain an equal quantity. Or you may multiply the breadth by 2, 3, or any other number, and divide the length by the same number that you multiplied the breadth by. Or lastly, if you set off a double proportion, that is $\frac{2}{6}$ or $\frac{1}{3}$ from C towards D, and from the point where it falls, draw a line to the angle A, you will have a triangle equal to $\frac{1}{6}$ of the square ACDN.

But to return to triangles, the most simple and primitive of all Rectilinears, and therefore, the most considerable in this case of partition, as giving laws often to the rest; it may fall out, that a triangle must be divided, by a line from some point in a side, so as that line may either be parallel to some other side, or not parallel to any.

Let

Let ABC be a triangle given, and let it be required to cut off $\frac{3}{5}$ by a line parallel to AB .

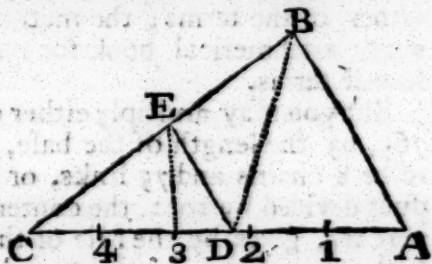
First, on the line AC describe the semi-circle AEC , on whose diameter, C A , divided into five equal parts according to the greater term, and upon three



of those parts, the lesser term, erect the perpendicular DE , which cuts the arch line in E , then set the length of the line CE from C , to 4, and from thence draw the line 4 G parallel to AB ; so will the triangle CG 4 contain $\frac{3}{5}$ of the triangle ABC , as was required.

Now, for the latter case, when the line of partition goes not parallel with any side, take this example.

Let ABC be a triangle given to be divided into two parts which shall be in proportion to one another, as 3 is to 2, by a line drawn from the point D in the base, or line AC .



From the limited point D , draw a line to the angle B ; then divide the base AC into five equal parts, and from the third point of the division draw a line to E parallel to BD . Lastly, from E draw the line ED , so shall the trapezium $ABED$ be in content as 3 to 2, to the new triangle DEC .

Before we have done with the division of triangles, it will be necessary to add the three following cautions.

1. You must be sure to take very exactly the distance of every point, where a division line cutteth any side; to one of the ends of the same side, as in this last figure, the distances BE and AD ; which distances being

ing applied to the scale by which the triangle was protracted, will shew at how many chains and links end you are to make your division or line on the field itself.

2. The proportions by which you are to divide, are not always so formally given as in the former examples, but are sometimes to be found out by arithmetical working; as in this case.

Suppose a triangular field of 6 acres, 2 roods, and 31 perches, must be divided so that one of the two parts shall be 4 acres, 3 roods, and 5 perches, and the other consequently 1 acre, 3 roods, and 26 perches; reduce both measures into perches, and the one will be 705, and the other 306. Their sum is 1071, which by their common measure, being reduced into their lowest terms of proportion in whole numbers, will be 5, 2, and 7, which shews that the triangle being divided into 7 equal parts, the one must have 5 of those 7 parts, and the other 2. And observe, that it will be sufficient to find the common measure between the sum of your terms, and either of the terms; the method whereof is shewn in every arithmetical book for reducing fractions to their lowest terms.

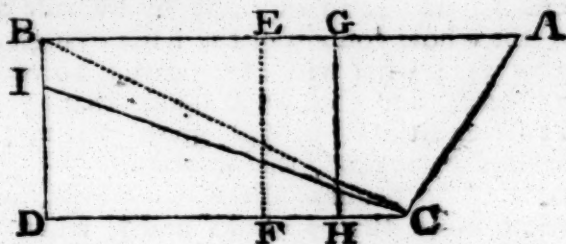
But you may multiply either of the parts; as suppose 765, by the length of the base, which we will suppose to be 8 chains and 75 links, or 87 links; and that product divided by 1071, the content of the whole close in perches, gives by the rule of three direct, 625 links, or 6 chains and 1 pole, the true distance from either end of the base, that your mind or occasions may direct you to the point of division; for the division must not only be for proportion or quantity, but also as to position or situation of parts upon the paper, as it is required to be on the ground.

3. In these and all other divisions of land, where a strict proportion in quantity is to be observed, you must have a respect to the rules given hereafter concerning the measuring of uneven ground, especially if one part prove much more uneven than another; and if there be any useful pond or well, you must draw your line of divisions through it; but if it be an useless pond, lake, or puddle, or if there be any boggy or barren ground, that

that must be cast out in the divisions ; measure that first, and subtract it from the content of the whole close, and then lay the just proportion of the remainder on that side that is free from it, that the other may have its just part also, besides that which is useless.

What hath been said, with an ordinary measure of discretion, may sufficiently instruct a young artist to divide triangles, Parallelograms, and regular Polygons, in an artificial way ; but because many closes and open grounds are Trapezias, and many irregular Polygons, and even those that are regular enough, may fall under an irregular division, in regard of the quality of the land, woods upon it, or quarries in it, or the conveniences of ways, currents of water, situation in respect to adjacent lands, &c. we shall propose a method, which though it has somewhat of the mechanic in it, will be singularly useful in such cases.

Let $ABDC$ be a Trapezium to be divided betwixt a young heir and his mother, so that



his part may be double to her's. Having by a Diagonal BC divided it into two triangles, you will find the content of the triangle ABC to be 138562 square links, and the triangle BCD to contain 103468, in all 242020 square links, which, if reduced, as hath been formerly taught, would amount to 2 acres, 1 rood, and 27 perches.

But for the present work, they are in a better order already. Dividing then 242020 into three parts, each of them is 80676 and $\frac{2}{3}$, two thirds of these third parts must contain 161353 $\frac{1}{3}$; but the fraction $\frac{1}{3}$ being inconsiderable, regard it not.

Then if you resolve to lay out the double part towards the line BD , strike at a venture the line EF , and measure the trapezium bounded by that line, and the opposite side BD , together with the interjacent parts of the lines AB and CD , which you will find to contain

tain 119140 square links; but because it should have been 161353, subtract 11940 out of 161353, and their difference is 42213, and perceiving that the lines AB and CD are very nearly parallel, and finding their distances where they are cut by the line EF, to be 326 links, or 3 chains and 26 links, divide 42213 by 326, and the quotient is 129 links and almost a half, at which distance draw the line GH parallel to EF, so shall the trapezium GBDH be the heir's part.

Another way whereby the same thing may be performed is this. Finding the triangle ABC to contain 138650 square links, subtract it out of the heir's part, viz. 161353, the difference 22803, shews how many square links must be taken out of the triangle BCD, and added to the ABC; which to perform with all necessary exactness, suppose the side or line BD to be the base, which by measure proves to be 344 links, or 3 chains, and 44 links. Say by the rule of three direct, if the whole content of the lesser triangle, viz. 103468, give 344; what will 22803 give? So will the result be 75 links, and somewhat more than $\frac{1}{4}$ of a link, for 22803 multiplied by 344, gives 7844232, which being divided by 103468, the quotient is $75 \frac{182132}{103468}$, or according to decimal division, 75.8131, which is somewhat more than 75 links and $\frac{1}{4}$, wherefore extending your compasses upon the scale to almost 76 links, set that distance upon the line BD, from B to I, and draw the line CI; so shall the trapezium ABIC be double to the triangle ICD, within so small a matter, as is not worth regarding, though the land were a rich meadow.

Concerning the boundaries of land, or where the lines to be measured, must begin and end.

If there be no agreement between the parties concerned, (for if there be, that must be observed) reason and custom are the Surveyor's guide.

The farmer speaks loudly, that when a piece of arable or meadow land is to be sown or mown, no more should be measured, nor expected to be paid for, either to the letter or workmen, than the plough or scythe can

go over. So also when a parcel of land is let for pasture to a farmer, it seems very reasonable that all, and only so much should be measured, as is useful for that purpose.

But commons to be inclosed, are usually measured, except it be otherwise agreed, to the uttermost bounds of every man's particular proportion, without any allowance for ditch or fence, every man being to make them upon his own of what breadth he pleases. Nor is this unreasonable, for it is as good for one as another, and the rate paid to the Lord is usually very small, and sometimes nothing.

It is also very usual in measuring betwixt lord and tenant, in case of leases for lives or long terms of years, to extend the lines to the utmost bounds of the tenant's claim, taking in the walls, hedges, and ditches; but this is accounted very hard, and frequently proves very unequal among the tenants of a lordship; some being forced to make much more waste of their ground this way, than others that hold as much or more. But where the custom obtains, the surveyor must observe it; for it is the business of others to appoint what must be measured, and he is only to measure truly what is so appointed.

Lastly, in case of a sale by measure, at a rate agreed upon by each, no boundaries being specified in the bargain, the rule is, to extend the lines to the quick-wood-row, that is, to the place where the quick-wood actually grows, or where, according to the general rule, it ought to be set.

A description of the plain table, the protractor, and lines of chords.

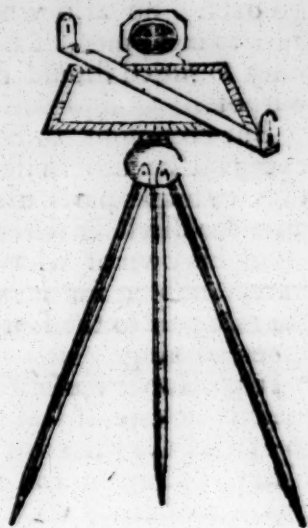
Though what hath been already said, may competently suffice to instruct any young artist in measuring a close of land, yet to advance him a degree higher in useful knowledge, we shall take occasion to describe to him the plain table, which consists of several parts.

The

The table itself is a Parallelogram of oak or any other wood, about fifteen inches long, and twelve broad, consisting of two several boards, round which are ledges of the same wood; the two opposite of which being taken off, and the spangle unscrewed from the bottom, the aforesaid two boards may be taken asunder for ease and convenience of carriage. For the binding of the two boards and ledges fast, when the table is set together, there is a box-jointed frame, about three quarters of an inch broad, and of the same thickness as the boards, which may be folded together in six pieces. This frame is so contrived, that it may be taken off and put on the table at pleasure, and may go easily on the table either side being upwards. This frame also is to fasten a sheet of paper upon the table, by forcing down the frame, and squeezing all the edges of the paper, so that it lies firm and even upon the table, that thereby the plot of a field or other inclosure, may conveniently be drawn upon it.

On both sides this frame, near the inward edge, are scales of inches subdivided into ten equal parts, having their proper figures set to them. The uses of these scales of inches are for the ready drawing of parallel lines upon the paper; and also for shifting your paper, where one sheet will not hold the whole work.

Upon one side of the said box-frame, are projected the 360 degrees of a circle, from a brass center hole in the middle of the table. Each of these degrees are subdivided into thirty minutes; to every tenth degree are set two numbers, one expressing the proper numbers of degrees, and the other the complement of that number of degrees to 360. This is done to avoid the trouble of subtraction in taking of angles.



On the other side of this frame, are projected the 180 degrees of a semi-circle from a brass center hole, in the middle of the table's length, and about a fourth part of its breadth. Each of these degrees are subdivided into thirty minutes; to every tenth degree are set likewise, as on the other side, two numbers; one expressing the proper number of degrees, and the other the complement of that number of degrees to 180; for the same reason as before.

The manner of projecting the degrees on the aforesaid frame is, by having a large circle divided into degrees, and every degree into thirty minutes; for then placing either of the brass center holes on the table, in the center of that circle so divided, and laying a ruler from that center to the degrees on the limb of the circle, where the edge of the ruler cuts the frame, make marks for the correspondent degrees on the frame.

The degrees thus inserted on the frame, are of excellent use in wet or stormy weather, when you cannot keep a sheet of paper on the table. These degrees will also make the plain-table a Theodolite, or semi-circle, according to which side of the frame is uppermost.

There is a box, with a needle and card, covered with a glass, fixed to one of the long sides of the table, by means of a screw, that it may be easily taken off. This box and needle is very useful for placing the instrument in the same position upon every remove.

There belongs to this instrument a brass socket and spangle, screwed with three screws to the bottom of the table, into which must be put the head of the three-legged staff, which may be screwed fast by means of a screw in the side of the socket.

There is also an index belonging to the table, which is a large brass-ruler, at least sixteen inches long, and two inches broad, and so thick as to make it strong and firm having a sloped edge called the fiducial edge, and two sights screwed perpendicularly on it of the same height. They must be set on the ruler exactly at the same distance from the fiducial edge. Upon
this

this index it is usual to have many scales of equal parts, as also Diagonals and lines of chords.

When you would make your table fit for use, lay the two boards together, and also the ledges at the ends in their due places, according as they are marked. Then lay a sheet of white paper all over the table, which must be stretched over the boards, by putting on the box-frame which binds both the paper to the boards, and the boards to one another: then screw the socket on the back-side of the table, and also the box and needle in its due place, the meridian line of the card lying parallel to the meridian of the diameter of the table; which diameter is a right line drawn upon the table from the beginning of the degrees through the center, and so to the end of the degrees. Then put the socket upon the head of the staff, and there screw it. Put also the sights upon the index, and lay the index upon the table. So is your instrument prepared for use, as a plain table, Theodolite, or semi-circle.

The Protractor is an instrument so well known, and so easy to be made and procured, that we shall be very brief in the description of it. As it is usually made, it consists of two parts, a scale and a semi-circle; the scale is however no necessary part of it, but serves if you are not otherwise provided, for uses before-mentioned in the case of the plain scales.

But the semi-circle is more essential. This may be made of brass, or other metal, and of any convenient size, as four inches, more or less, for the straight side; this semi-circle being bounded as all others are by two lines; the one right or straight, the other circular.

The right line is divided precisely in the middle, by a point which is in the center, upon which the circular boundary is drawn, and two other arches concentric with it.

The center, when the semi-circle goes alone without the scale, should be guarded with two lips, one on each side, or a little loup, for the conveniency of turning the instrument about a pin fixed in a paper.

The

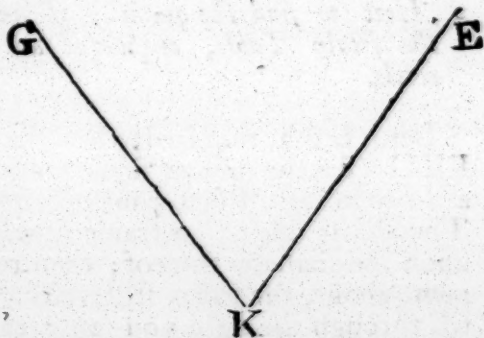
The arched or circular edge is divided into 180 degrees, or equal parts, numbered by tens, upon the upper concentric arch from 0 to 180, and in the lower from 180 to 360. So that by applying the straight edge of the Protractor twice to any line, keeping the center right upon a pin fixed in the line, that is, with the semi-circle first above it, and then below it, or contrarily, you may draw a whole circle by the guidance of the arch, or set out any number of degrees, as will appear more plainly hereafter.

A line of chords is a line divided into 90 unequal parts, whereof 60, and the Radius upon which the circle was drawn, are equal, and the divisions upon that line are equal to the next extent in a right line of so many degrees from the beginning of the quadrant, as answer thereto.

When lines of chords are cut upon wood, it is both usual and necessary that there be two studs of brass, the one at the beginning, and the other at 60 degrees, with little Holes for the feet of the compasses, when you take the extent of the Radius, to preserve the line from being wounded by the compasses, and thus fenced, it will, in case of necessity, do the work of a Protractor, but not altogether so commodiously.

1. *How to observe an angle in the field by the plain table.*

Suppose E, K, G, to be two hedges, or two sides of a field, including the angle EKG, and it is required to draw upon the table an angle equal thereto: First place your



instrument as near the angular point K as conveniency will permit, turning it about till the north end of the needle hang directly over the meridian line in the card, and then screw the table fast. Then upon your paper, will

with your protracting pin, which is a fine needle put into a piece of box, or ivory, neatly turned, or compass-point, assign any point at pleasure upon the table, and to that point apply the edge of the index, turning the index about upon that point, 'till through the sights you see a mark set up at E, or parallel to the line EK, and then with your protracting-pin, compass-point, or pencil, draw a line by the side of the index to the assigned point upon the table: then the table remaining immoveable, turn the index about upon the fore-mentioned point, and direct the sights to the mark set up at G, or parallel thereto, that is, so far distant from G, as your instrument is placed from K: and then by the side of the index, draw another line to the assigned point. Thus will there be drawn upon the table two lines representing the hedges EK, and KG, and which include an angle equal to the angle EKG. And though you know not the quantity of this angle, yet you may find it, if required: for in working by this instrument, it is sufficient only to give the proportions of angles, and not their quantities in degrees, as in working by the Theodolite, semi-circle, or circumferentor. Also in working by the plain table, there needs no protraction at all, for you will have upon your table the true figure of any angle or angles that you observe in the field in their true positions, without any further trouble.

2. *How to find the quantity of an angle in a field, by the Plain Table, considered as a Theodolite or semi-circle.*

Let it first be required to find the quantity of the angle EKG (see the above figure) by the plain table, as a Theodolite. Place your instrument at K, with the Theodolite side of the frame upwards, laying the index upon the diameter thereof; then turn the whole instrument about, the index still resting upon the diameter, till through the sight you espy the mark at E: then screwing the instrument fast there, turn the index about upon the Theodolite center hole, in the middle of the table, till thro' the sight you espy the mark at G. Then note what degrees on the frame of the table are cut by
the

the index, and those will be the quantity of the angle EKG sought.

You must proceed in the same manner for finding the quantity of an angle by the plain table as a semi-circle; only put the semi-circle side of the frame upwards, and move the index upon the other center hole.

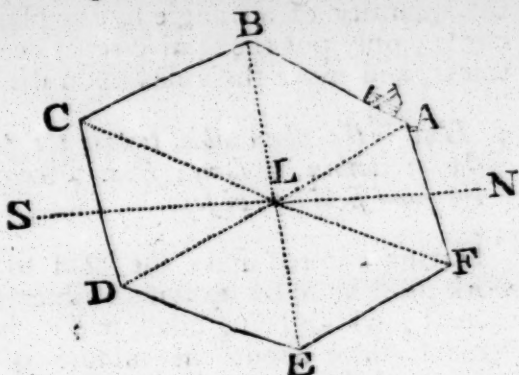
3. *How by the plain table, to take the plot of a field at one station within the same; from whence all the angles of the same field may be seen.*

Having entered upon the field to survey, your first work must be to set up some visible mark at each angle thereof; which being done, make choice of some convenient place about the middle of the field, from whence all the marks may be seen, and there place your table, covered with a sheet of paper, with the needle hanging directly over the meridian line of the card, which you must always have regard to, especially when you are to survey many fields-together. Then make a mark about the middle of the paper, to represent that part of the field where the table stands; and laying the index upon this point, direct your sights to the several angles where you before placed marks, and draw lines by the side of the index upon the paper. Then measure the distance of each of these marks from your table, and by your scale set the same distances upon the lines drawn upon the table, making small marks with your protracting-pin, or compass-point, at the end of each of them. Then lines being drawn from one to the other of these points, will give you the exact plot of the field; all the lines and angles upon the table being proportional to those of the field.

EXAMPLE.

EXAMPLE.

Suppose the plot of the field ABCDEF was to be taken. Having placed marks in the several angles thereof, make choice of some proper place about the middle of the field, as at L, from whence you can see all the marks before



placed in the several angles; and there place your table. Then turn your instrument about, till the needle hang over the meridian line of the card, denoted by the line NS.

Your table being thus placed with a sheet of paper thereon, make a mark about the middle of your table, which shall represent the place where your table stands. Then applying your index to this point, direct the sights to the first mark at A; and the index resting there, draw a line by the side of your index to the point L. Then with your chain measure the distance from L, the place where your table stands to A, the first mark, which suppose eight chains ten links. Then take eight chains ten links from any scale, and set that distance upon the line from L to A.

Then directing the sights to B, draw a line by the side of the index as before, and measure the distance from your table at L, to the mark at B, which suppose eight chains, seventy-five links. This distance taken from your scale, and being applied to your table from L to B, will give the point B, representing the second mark. Then direct the sights to the third mark C, and draw a line by the side of the index, measuring the distance from L to C, which suppose ten chains sixty-five links. This distance being taken from your scale, and

and applied to your table from L to C, will give the point C, representing the third mark.

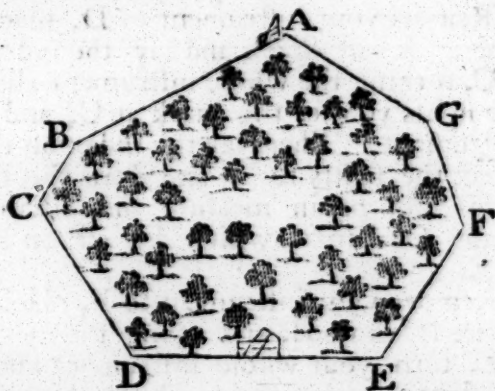
In this manner you must deal with the rest of the marks at D, E, and F, and more if the field had consisted of more sides and angles.

Lastly, when you have made observations of all the marks round the field, and found the points ABCDE and F upon your table, you must draw lines from one point to another, till you conclude where you first began; as draw a line from A to B, and from B to C, from C to D, from D to E, from E to F, and from F to A, where you began; then will ABCDEF be the exact figure of your field, and the line NS, the meridian.

4. *To take the plot of a wood, park, or other large level plain by the plain table, in measuring round the same.*

Suppose ABCDEFG to be a large wood, whose plot you desire to take upon the plain table.

Having put a sheet of paper on the table, place your instrument at the angle A, and direct your sights to the next angle at B, and by the side of the index draw a line upon your table, as the



line AB. Then measure by the hedge side, from the angle A to the angle B, which suppose twelve chains, five links. Then from your scale take twelve chains, five links, and lay off upon your table from A to B. Then turn the index about, and direct the sights to G, and

and draw the line AG upon the table. But at present you need not measure the distance.

Remove your instrument from A, and set up a mark where it last stood, and place your instrument at the second angle B. Then laying the index upon the line AB, turn the whole instrument about, till through the sights you see the mark set up at A, and there screw the instrument. Then laying the index upon the point B, direct your sights to the angle C, and draw the line BC upon your table. Then measuring the distance BC four chains, forty-five links, take that distance from your scale, and set it upon your table from B to C.

Remove your instrument from B, and set up a mark in the room of it, and place your instrument at C, laying the index upon the line CB; and turn the whole instrument about, till through the sights you espy the mark set up at B, and there fasten your instrument. Then laying the index on the point C, direct the sights to D, and draw upon the table the line CD. Then measure from C to D eight chains, eighty-five links, and set that distance upon your table from C to D.

Remove your instrument to D, placing a mark at C, where it last stood, and lay the index upon the line DC, turning the whole instrument about, till through the sights you see the mark at C, and there fasten the instrument. Then lay the index on the point D, and direct the sights to E, and draw the line DE. Then with your chain measure the distance DE thirteen chains four links, which lay off on the table from D to E.

Remove your instrument to E, placing a mark at D, where it last stood, and laying the index upon the line DE, turn your whole instrument about, till through the sights you see the mark at D, and there fasten the instrument. Then lay the index on the point E, and direct the sights to F, and draw the line EF. Then measure the distance EF, seven chains seventy links, which take from your scale, and lay off from E to F.

Remove your instrument to F, placing a mark at E, where it last stood, and lay the index upon the line EF, turning the instrument about, till you see the mark set up
at

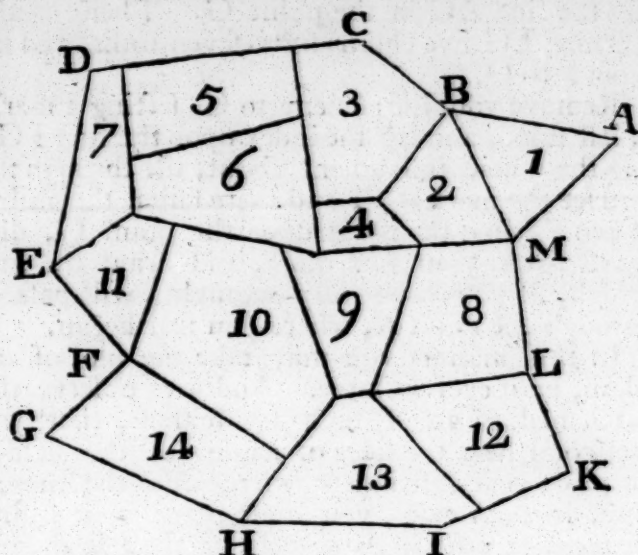
at E, and there fasten the instrument. Then laying the index on the point F, direct the sights to G, and draw the line FG upon the table, which line FG will cut the line AG in the point G. Then measure the distance FG five chains sixty-seven links, and lay it off from F to G.

Remove your instrument to G, setting a mark where it last stood, and lay the index upon the line FG, turning the whole instrument about, till through the sights you see the mark at F, and there fasten the instrument. Then laying the index upon the point G, direct the sights to A, your first mark, and draw the line GA, which, if you have truly wrought, will pass directly through the point A, where you first began.

In this manner you may take the plot of any level plain, be it ever so large. And here observe, that very often hedges are of such a thickness, that you cannot come near the sides or angles of the field, either to place your instrument, or measure the lines: therefore, in such cases, you must place your instrument, or measure your lines parallel to the side thereof; and then your work will be the same as if you measured the hedge itself. Take notice also, that in thus going about the field you may assist yourself greatly by the needle. For looking what degree of the card the needle cuts at one station, if you remove your instrument to the next station, and with your sights look to the mark where your instrument last stood, you will find your needle to cut the same degree again, which will give you no small satisfaction in the prosecution of your work. And though there be a hundred or more sides, the needle will still cut the same degrees at all of them, except where you have committed some former error; therefore, at every station have an eye to the needle.

Concerning the method of plotting many closes together, whether the ground be even or uneven.

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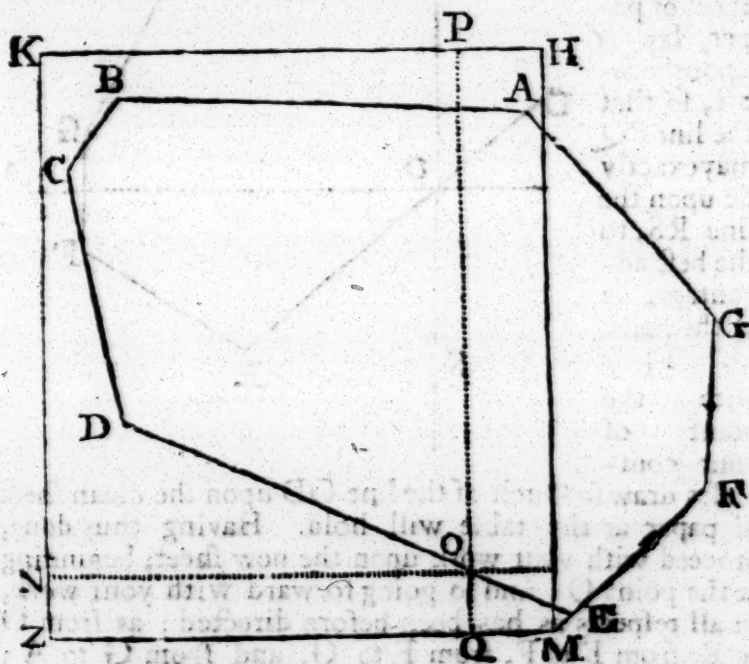
according to their several shapes and situation: First, draw the plot of the whole by the method of circulation, planting your instrument either at every angle, or only at the most considerable, either within or without, as you find most convenient. This being done, a line from B to M, gives the triangle ABM for the first close. In the next place go round the second close, beginning at M, then to B, and so about to M again; then for the third close, plant your table at C, and go round to B, the line CB being already protracted, and so of all the rest, still observing which are common lines belonging to several closes, representing the fences, that you may avoid the trouble of measuring those lines oftner than once, and lay every part of every close in its due place; and be sure to keep the instrument throughout the whole work in its true position by needle, fore-sight, and back-sight.

There are divers other ways of performing this work, but none more sure or plain, especially if the ground be uneven; for in that case, if you protract accord-

according to the length of lines measured from your station to the angles, you will put your closes into unproportionable shapes, except you reduce hypotenusal lines to horizontal, by instruments, or otherwise, which is somewhat troublesome; and the like may be said when you plot with the chain only.

Concerning shifting the Paper.

In taking the plot of a field by the plain table, and going about the same, as before directed, it may so happen when the field is remarkably large, or when you are to take many inclosures together, that the sheet of paper on the table will not hold all the work. In this case you must be obliged to take off that sheet, and put another clean one in the room of it; and in plotting a manor or lordship, many sheets may be thus changed which we call shifting the paper. To do this, the following method must be observed.

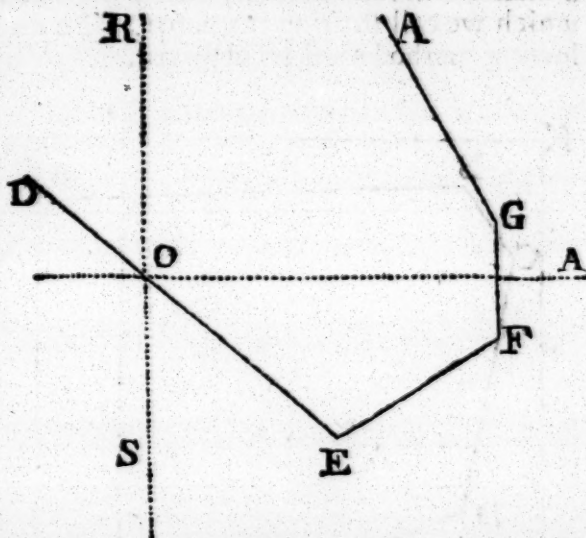


Suppose in going about to take the plot ABCDEFG as before directed, that you make choice of the angle
H at

at A for the place of beginning, and proceed from thence to B, from B to C, and from C to D; when you come to the angle at D, and are to draw DE, you want room to draw the same upon the table; do thus:

First, Through the point D draw the line DO, which is almost as much of the line DE as the table will contain. Then near the edge of the table HM, draw a line parallel to HM, by means of inches and subdivisions on the opposite sides of the frame, as PQ, and another line at right angles to that through the point O, as RS. This being done, mark this sheet of paper with the figure (1) about the middle thereof, for the first sheet. Then, taking this sheet off your table, put another clean sheet thereon, and draw upon it a line parallel to the contrary edge of the table, as the line RS.

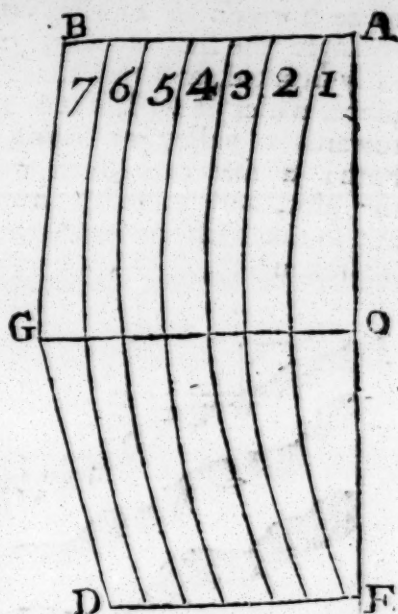
Then taking your first sheet of paper, lay it upon the table, so that the line PQ may exactly lie upon the line RS, to the best advantage, as at the point O. Then with the point of your compasses



draw so much of the line OD upon the clean sheet of paper as the table will hold. Having thus done, proceed with your work upon the new sheet, beginning at the point O; and so going forward with your work, in all respects as has been before directed; as from O to E, from E to F, from F to G, and from G to A; shifting your paper as you have occasion.

Concerning the plotting of a town-field, where the several lands, butts, or doles, are very crooked; with an observation concerning hypotenusal, or sloping boundaries.

Suppose ABGDFO divided, in the manner of a common field, into parts or doles, belonging to seven several men: first plot the whole, as hath been before directed; then measuring from A to B upon the land, note down, as you go along, at how many chains or links, or both, the division is between dole and dole, and accordingly mark them out by the help of scale and compasses on the line AB on the paper-plot. In the very same manner



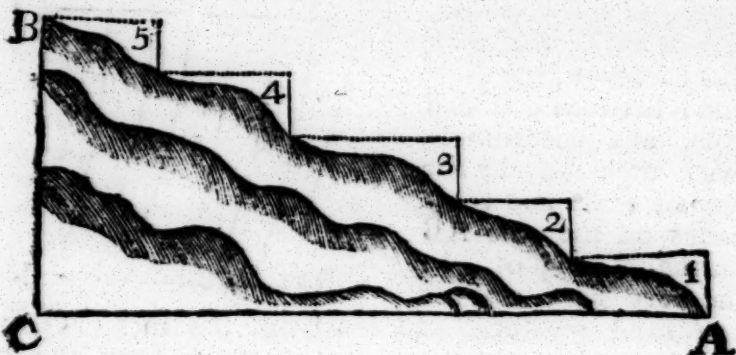
you must measure, and mark out the lines OG and FD; which being done, take the paper from the instrument, and laying it before you on a table, with the side AF towards you, the compasses must be so placed, that one foot resting upon the table, the other may pass through the points of division upon all the three lines, viz. AB, OG, and FD, as they do in the above figure.

If the content of any one or more of these parts, butts, or doles, be desired without plotting, it may easily be found, by your plain table, in the manner following.

Take the breadth by your chain at the head and lower end, and adding those numbers together, the third part of the sum is the equated breadth; by which multiplying the length measured down the edge, or middle, the product gives the content.

But both in this case and that mentioned before, the figure of a plot may be somewhat disordered, not only

the unevenness of the ground within, but also by the greater declivity of the ground where the boundary lines pass, either of the whole plot, or particular parcels. For whereas in plotting, every line is presumed to be horizontal, or level, that it may pass from angle to angle the shortest way, and that every part may be duly situated, and not thrust another out of its right place, which it will, if it be not level, but falling down towards a valley or rising up a hill, or perhaps both; a line over such ground will be false as to the plot, and therefore must be reduced to a level, and so taken off the scale, and protracted in the following manner:



Suppose ABC to be part of a hill falling within your plot, your boundary line going crooked from A to B, following the surface of the ground. To find the horizontal line, equal to AC, cause some person to stand at the point A, the foot of the hill, and to hold up the end of the chain to a convenient height, and gently ascending the hill, you must draw it level, and make a mark where it toucheth the hill, observing the number of links between your assistant's hand, and that place where he must take his second standing, and hold it up as before, and so draw it out level again, 'till it touch the place where he must take his third standing, noting the links as before; and so proceed, 'till at last, from his fifth standing, you draw the chain level to the highest point within your plot, viz. the point B. And as the dotted lines of this figure were evidently equal to the line AC, so are the links noted down at every station, when summed

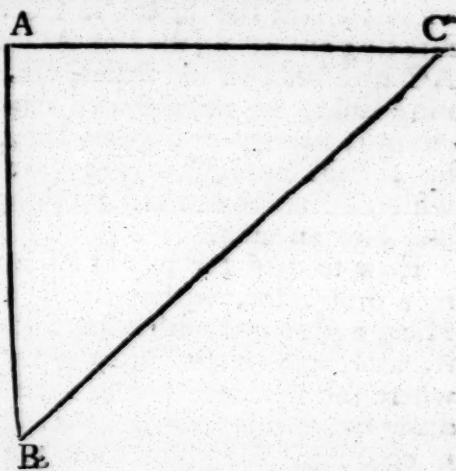
med up, equal to the horizontal line of that part of the hill.

In the very same manner, only inverting the order, you may find the horizontal lines going down hill, where that is most convenient; and if there be both ascents and descents in one line betwixt two angles, the horizontal lines of both must be found and joined together without protraction.

All that has been said concerning inequalities of rising or falling ground, is to be understood when they are considerable, and a very exact plot required; for small ones, especially when much exactness is not expected, are not to be attended to.

Concerning taking inaccessible distances by the plain table, and accessible altitudes by the protractor.

In order to answer questions of this kind, it will be necessary to measure the distance between two stations, properly chosen on the banks of the river, from whence some object on the opposite bank may be seen; then take the angles at each station, and measure very accurately the distance between



them; these observations being plotted, the distance may be easily measured.

EXAMPLE.

Let AC be the unknown breadth of a river, over which the General, that he may inform himself how many boats and planks are necessary to be brought, commands you to tell him the true distance from A, where he is at present to C, a little boat-house on the other side the water.

H₃

To

To fatisfy his demand, plant your table covered with paper at A, caufing a mark to be fet up at B, at a good diftance from you, along the bank of the river, the further the better, if the diftance does not hinder the fight; then having chofen a point to represent A, and laid the fiducial edge upon it, direct your fights towards C and B, and ftrike lines towards them, which done fet up a mark at A, and from thence meafure to B, fix chains, thirty-two links, and plant your instrument at B, laying the fiducial edge to the line AB, and turning about the head of the instrument upon the ftaff, till through the fights you fee the mark at A, and then fcrew it faft.

In the laft place take fix chains, thirty-two links off your fcale, and fet it on the line AB, from A to B, and laying the fiducial edge to the point B, from thence direct the fights to C, and draw the line BC, meeting or cutting the line AC in C: fo fhall the fpace AC meafured on the fcales, viz. eight chains, twenty-nine links, be the diftance required; and becaufe the chain is twenty-two yards long, if you multiply 8.29 by 22, the product is 182 yards, and $\frac{38}{100}$ of a yard, which by reduction is fomething more than 13 inches and $\frac{2}{3}$ of an inch.

How to take the height of a tree, tower, or fteeple by a protractor, without any arithmetical operation. Hang a plummet with a fine filk thread to the center of it, and hold it ftedfaftly with that end to your eye where the numbers begin, then look ftreight along the diameter, ftill removing backward and forward as there is occafion, till you fee the top of the tree, tower, or fteeple, and the thread at the fame time fall upon forty-five degrees, fo fhall the diftance from your eye to the tree, tower, or fteeple, meafured in an horizontal or level line, together with the height of your eye above the bottom of it, be equal to the height of the object.

If either for convenience of fight, or any other reafon you think proper to place the other end of the diameter to your eye, then the thread for the above trial muft fall upon 135 deg. inftead of 45.

FIG.

FIG. VI.

To measure an accessible height by means of a plain Mirror.

Let AB be the height to be measured; let the mirror be placed at C , in the horizontal plane BD , at a known distance BC ; let the observer go back to D , till he see the image of the summit in the mirror, at a certain point of it, which he must dili-



gently mark; and let DE be the height of the observer's eye. The triangles ABC and EDC are æquian-gular; for the angles at D and B are right angles; and ACB, ECD, are equal, being the angles of incidence and reflection of the ray AC, as is demonstrated in optics; wherefore the remaining angles at A, and E, are also equal: Therefore, by 4th, 6. Eucl. it will be, as CD to DE, so CB to BA; that is, as the distance of the observer from the point of the mirror in the right line betwixt the observer and the height, is to the height of the observer's eye, so is the distance of the tower from that point of the mirror, to the height of the tower sought; which therefore will be found by the rule of three.

Note 1. The observer will be more exact, if, at the point D, a staff be placed in the ground perpendicularly, over the top of which the observer may see a point of the glass exactly in a line betwixt him and the tower.

Note 2. In place of a mirror may be used the surface of water contained in a vessel, which naturally becomes parallel to the horizon.

FIG. VII.

To measure an accessible height AB by means of two staffs.

Let there be placed perpendicularly in the ground a longer staff DE, likewise a shorter one FG, so as the
H 4 observer

H. 4

observer:

observer may see A, the top of the height to be measured over the ends D, F, of the two staves; let FH and DC, parallel to the horizon, meet DE and AB in H and C; then the triangles FHD, DCA, shall be æquiangular; for the angles at C and H are right ones; likewise the angle A, is equal to the angle FDH, by 29.

1. *Eucl.*; wherefore the remaining

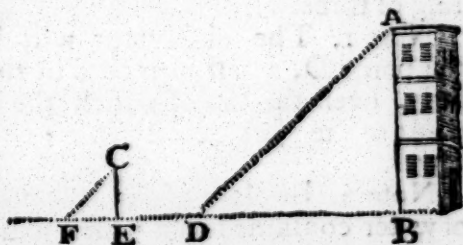
angles DFH, and ADC, are also equal: wherefore by 4. 6. *Eucl.* as FH, the distance of the staves, to HD, the excess of the longer staff above the shorter; so is DC, the distance of the longer staff from the tower, to CA, the excess of the height of the tower above the longer staff. And thence CA will be found by the rule of three.

To which if the length DE be added, you will have the whole height of the tower BA.



SCHOLIUM.

Many other methods may be occasionally contrived for measuring an accessible height. For example, from the given length of the shadow BD, I find out the height AB, thus: Let there be erected a staff CE perpendicularly, producing the shadow EF: The triangles ABD, CEF, are æquiangular; for the angles at B, and E, are right; and the angles ADB, CFE are equal each being equal to the angle of the sun's



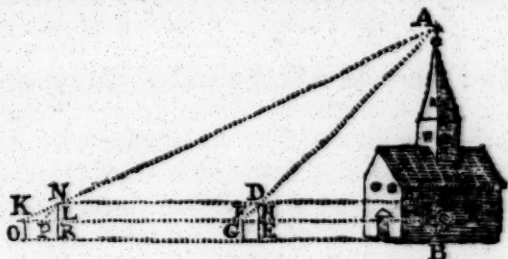
elevation above the horizon: therefore, by 4th, 6. *Eucl.* as EF the shadow of the staff, to EC the staff itself, so BD the shadow of the tower, to BA the height of the tower. Though the plane on which the shadow of the tower falls be not parallel to the horizon, if the staff be erected in the same plane, the rule will be the same.

To

To measure an inaccessible height by means of two staffs.

Hitherto we have supposed the height to be accessible, or that we can come at the lower end of it; now if, because of some impediment, we cannot get to a tower, or if the point whose height is to be found out, be the summit of a hill, so that the perpendicular be hid within the hill; if, I say, for want of better instruments, such an inaccessible height is to be measured by means of two staffs, let the first observation be made with the staffs DE and FG, as in prop. I. then the observer is to go off in a direct line from the height and first station, till he come to the second station; where he is to place the longer staff perpendicularly at RN,

and the shorter staff at KO, so that the summit A may be seen along their tops; that is, so that the points KNA may be in the same right line.



Thro' the point N let there be drawn the right line NP parallel to FA: wherefore in the triangles KNP, KAF, the angles KNP, KAF are equal, by the 29. 1. *Eucl.* also the angle AKF is common to both; consequently the remaining angle KPN, is equal to the remaining angle KFA. And therefore, by 4th, 6. *Eucl.* $PN : FA :: KP : KF$. But the triangles PNL, FAS are similar; therefore, by 4th, 6. *Eucl.* $PN : FA :: NL : SA$. Therefore, by the 11. 5. *Eucl.* $KP : KF :: NL : SA$. Thence, alternately; it will be, as KP (the excess of the greater distance of the short staff from the long one above its lesser distance from it) to NL, the excess of the longer staff, above the shorter; so KF, the distance of the two stations of the shorter staff, to SA, the excess of the height sought above the height of the shorter staff. Wherefore SA will be found by the rule of three. To which let the height of the shorter staff be added, and the sum will give the whole inaccessible height BA.

Note

Note 1. In the same manner may an inaccessible height be found by a geometrical square, or by a plain speculum. But we shall leave the rules to be found out by the student for his own exercise.

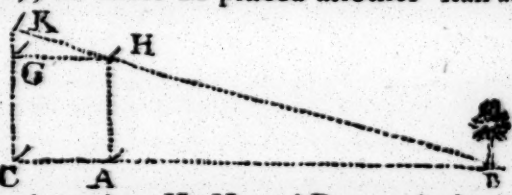
Note 2. That by the height of the staff we understand its height above the ground in which it is fixed.

Note 3. Hence depends the method of using other instruments invented by Geometricians; for example, of the Geometrical cross: and if all things be justly weighed, a like rule will serve for it as here. But we incline to touch only upon what is most material.

FIG. IX.

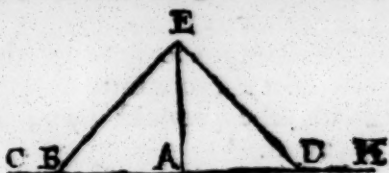
To measure the distance AB, to one of whose extremities we have access, by the help of four staffs.

Let there be a staff fixed at the point A; then going back at some sensible distance in the same right line, let another be fixed in C, so as that both the points A and B be covered and hid by the staff C; likewise going off in a perpendicular from the right line CB, at the point A, (the method of doing which shall be shown in the following *Scholium*), let there be placed another staff at H; and in the rightline CKG (perpendicular to the same CB, at the point C), and at the point C of it K, such that the points K, H, and B, may be in the same right line, let there be fixed a fourth staff. Let there be drawn, or let there be supposed to be drawn, a right line HG parallel to CA. The triangles KHG, HAB will be æquiangular; for the angles HAB, KGH are right angles. Also, by 29th, 1. *Eucl.* the angles ABH, KHG are equal; wherefore, by the 4th, 6. *Eucl.* as KG (the excess of CK above AH) to GH, or to CA, the distance betwixt the first and second staff; so is AH, the distance betwixt the first and third staff, to AB the distance sought.



S C H O L I U M.

To draw on a plane right line AE perpendicular to CH, from a given point A; take the right lines AB, AD, on each side equal; and in the points B and D, let there be fixed stakes, to which let there be tied two equal ropes BE, DE, (or one having a mark in the middle, and holding in your hand their extremities joined, (or the mark in the middle, if it be but one), draw out the ropes on the ground; and then, where the two ropes meet, or at the mark, when by it the rope is fully stretched, let there be placed a third stake at E; the right line AE will be perpendicular to CH in the point A, by 11th, 1. *Eucl.* In a manner not unlike to this may any problems that are resolved by the square and compasses, be done by ropes and a cord turned round as a radius.



C H A P. II.

Of measuring BRICKLAYER'S WORK.

Discipulus. **I**S there any difference between measuring bricklayer's work, and measuring superficies in general?

Philo. There is no difference in finding the content; but there is a very material difference in taking the dimensions, and finding the true value of any piece of work; because walls are of different thickness; sometimes ornamented, sometimes not; it will therefore be necessary to explain these particulars; for otherwise it will be impossible to measure truly the several particulars of bricklayer's work.

Discip. How are the dimensions taken?

Philo. The dimensions in brick-work, are estimated by the rod, consisting of 16 feet and a half, therefore the result is in square rods of $272\frac{1}{4}$ square feet each.

Hence, in practice, they divide the area found in square feet by 272, and the quotient will be the rods required.

Bricklayers always compute or value their work at the rate of a brick and an half thick ; and if the thickness of the wall happen to be more or less than that standard, it must be reduced to the above thickness, in the following manner.

Multiply the area of the wall, by the number of half bricks, contained in the thickness of the wall ; divide the product by 3, and the quotient is the area required.

Brick walls are sometimes wrought part of the way two inches thicker than the rest of the work ; these two inches serving as a water-table to the wall, which is commonly set off about two feet above the ground ; the brick-work may therefore be measured at the same thickness, which is above the water-table ; and then the two-inch-work may be measured, and afterwards reduced to that of a brick and a half.

Discip. I now sufficiently understand why there is a necessity for dividing general mensuration of superficies into different kinds : but you also mentioned something of ornamental work ; I should be glad to hear you explain what you meant by that expression ?

Philo. By ornamental mark is meant the arches over doors, windows, &c. Facias, Architraves, rubbed Returns, Friezes, Cornices, Rustices, &c. Piers, Columns, Pilasters, &c. which are valued by the square foot. But the bricklayers in general understand, by ornamental work, those particular parts in a building, which are ground, gauged, or polished. Others call carved mouldings, &c. by the name of ornamental work ; but this is generally undertaken at a certain price per foot, running measure.

Discip. I am greatly obliged to you, Sir, for your kind intelligence ; but should be glad to know how the dimensions of such works are taken.

Philo. When the content is to be found in square measure, the length is measured where it is greatest, and the height or breadth is measured by extending the line from the upper part of the surface, to what workmen call the Soffie, or under part of it, including

ing in this the bottom home to the back part of the bricks. The same method is observed in measuring Cornices, and Facias, &c. They are girt over all their surfaces, including both the upper and under returns, no part being omitted.

It is also necessary to observe, that when two walls join in an angle, they measure the length of one on the outside, and that of the other on the inside; the sum of these two mensurations, being considered as the true length of the whole wall.

In all buildings of any height, the thickness of the walls decreases as they rise; and this decrease consists generally of half a brick. The thickness is set off on the inside, and generally in a place where the floor will be laid; a contrivance by which the set-off is concealed. These different thicknesses therefore must be measured separately, their areas found, and then added together; for that sum gives the true content of the wall.

If a chimney stand by itself, without any party-wall being joined to it, they measure it in the following manner: they girt it about for the length, and reckon the height of the story for the breadth: The thickness must be the same with that of the jaumbs, provided the chimney be wrought upright from the mantle-tree to the cieling; not deducting any thing for the vacancy, between the mantle-tree and the hearth, because of the gathering of the breast and wings to make room for the hearth in the next story.

But when the chimney-back is a party-wall, and the wall measured by itself; they take the depth of the two jaumbs, and the length of the breast for the length; the height of the story for the breadth; and the thickness of the jaumbs for the proper thickness: With these dimensions the content of the chimney is easily found.

Chimney-shafts, or those parts which appear above the roof, are measured by girting them with a line about the least part for the length; and taking the height for the breadth, if they are four inches thick-
 1
 reckon

reckon the thickness at a brick and a half, in consideration of their plaistering, or as they call it, pargeting, the inside of the shaft, and the trouble of raising the scaffold. In some places, indeed, it is customary to allow double measure for chimnies.

When the chimney stands in one of the angles or corners of the room, they multiply half the product of the height into the breadth of the breast or front, and reckon the thickness by that of the jaumbs.

But you must always remember in measuring brick-work, that you must deduct the spaces left for doors, windows, &c.

Discip. I am highly obliged to you for this explanation, and perceive that the difficulty consists in taking the dimensions properly, and reducing the whole to a brick and a half thick; that is, by multiplying the area by the number of half bricks, and dividing the product by 3.

Philo. You are extremely right, Discipulus; and in order to shorten the latter part of the work, I mean, to save the trouble of multiplying the area by the number of half bricks in the wall, &c. and dividing that product by 3, I have calculated the following TABLE, which will save a great deal of trouble, and remove the danger of committing mistakes.

A TABLE

A
T A B L E

For reducing Brick-work of any thickness to the standard thickness of a brick and a half.

		$\frac{1}{2}$ Brick.	1 Brick.	$1\frac{1}{2}$ Brick.	2 Bricks.
		R. Q. F.	R. Q. F.	R. Q. F.	R. Q. F.
1 Quarter.		0 0 22	0 0 45	0 1 0	0 1 22
2 Quarters.		0 0 45	0 1 22	0 2 0	0 2 45
3 Quarters.		0 2 0	0 2 0	0 3 0	1 0 0
Rods contained in the superficial measure of the wall.	1	0 1 22	0 2 45	1 0 0	1 0 22
	2	0 2 45	1 1 22	2 0 0	2 1 45
	3	1 0 0	2 0 0	3 0 0	4 0 0
	4	1 1 22	2 2 45	4 0 0	5 1 22
	5	1 2 45	3 1 22	5 0 0	6 2 45
	6	2 0 0	4 0 0	6 0 0	8 0 0
	7	2 1 22	4 2 45	7 0 0	9 1 22
	8	2 2 45	5 1 22	8 0 0	10 2 45
	9	3 0 0	6 0 0	9 0 0	12 0 0
	10	3 1 22	6 2 45	10 0 0	13 1 22
	11	3 2 45	7 1 22	11 0 0	14 2 45
	12	4 0 0	8 0 0	12 0 0	16 0 0
	13	4 1 22	8 2 45	13 0 0	17 1 22
	14	4 2 45	9 1 22	14 0 0	18 2 45
	15	5 0 0	10 0 0	15 0 0	20 0 0
	16	5 1 22	10 2 45	16 0 0	21 1 22
	17	5 2 45	11 1 22	17 0 0	22 2 45
	18	6 0 0	12 0 0	18 0 0	24 0 0
	19	6 1 22	12 2 45	19 0 0	25 1 22
	20	6 2 45	13 1 22	20 0 0	27 2 45
	21	7 0 0	14 0 0	21 0 0	28 0 0

A TABLE

T A B L E

For reducing Brick work of any thickness to the standard thickness of one brick and a half.

		2½ Bricks.			3 Bricks.			3½ Bricks.			4 Bricks.		
		R.	Q.	F.	R.	Q.	F.	R.	Q.	F.	R.	Q.	F.
1	Quarter.	0	1	45	0	2	0	0	2	22	0	2	45
2	Quarters.	0	3	22	1	0	0	1	0	45	1	1	22
3	Quarters.	1	1	0	1	2	0	1	3	0	2	0	0
Rods contained in the superficial measure of the wall.	1	1	2	45	2	0	0	2	1	22	2	2	45
	2	3	1	22	4	0	0	4	2	45	5	1	22
	3	5	0	0	6	0	0	7	0	0	8	0	0
	4	6	2	45	8	0	0	9	1	22	10	2	45
	5	8	1	22	10	0	0	11	2	45	12	1	22
	6	10	0	0	11	0	0	14	0	0	16	0	0
	7	11	2	45	14	0	0	16	1	22	18	2	45
	8	13	1	22	16	0	0	18	2	45	21	1	22
	9	15	0	0	18	0	0	21	0	0	24	0	0
	10	16	2	45	20	0	0	23	1	22	26	2	45
	11	18	1	22	22	0	0	25	2	45	29	1	22
	12	20	0	0	24	0	0	28	0	0	32	0	0
	13	21	2	45	26	0	0	30	1	22	34	2	45
	14	23	1	22	28	0	0	32	2	45	37	1	22
	15	25	0	0	30	0	0	35	0	0	40	0	0
	16	26	2	45	32	0	0	37	1	22	42	2	45
	17	28	1	22	34	0	0	39	2	45	45	1	22
	18	30	0	0	36	0	0	42	0	0	48	0	0
	19	31	2	45	38	0	0	44	1	22	50	2	45
	20	33	1	22	40	0	0	46	2	45	53	1	22
	21	35	0	0	42	0	0	49	0	0	56	0	0

A TABLE

A

T A B L E

For reducing Brick-work of any thickness to the standard thickness of a brick and a half.

		$4\frac{1}{2}$ Bricks			5 Bricks			$5\frac{1}{2}$ Bricks		
		R.	Q.	F.	R.	Q.	F.	R.	Q.	F.
1	Quarter.	0	2	0	0	5	22	0	3	45
2	Quarters.	1	3	0	1	2	45	1	3	22
3	Quarters.	2	1	0	2	2	0	2	3	0
Rods contained in the superficial measure of the wall.	1	3	0	0	3	1	22	3	2	45
	2	6	0	0	6	2	45	7	1	22
	3	9	0	0	10	0	0	11	0	0
	4	12	0	0	13	1	22	14	2	45
	5	15	0	0	16	2	45	18	1	22
	6	18	0	0	20	0	0	22	0	3
	7	21	0	0	23	1	22	25	2	45
	8	24	0	0	26	2	45	29	1	22
	9	27	0	0	30	0	0	33	0	0
	10	30	0	0	33	1	22	36	2	45
	11	33	0	0	36	2	45	40	1	22
	12	36	0	0	40	6	0	44	0	0
	13	39	0	0	43	1	22	47	2	45
	14	42	0	0	46	2	45	51	1	22
	15	45	0	0	50	0	0	55	0	0
	16	48	0	0	53	1	22	58	2	45
	17	51	0	0	56	2	45	62	1	22
	18	54	0	0	60	0	0	66	0	0
	19	57	0	0	63	1	22	69	2	45
	20	60	0	0	66	2	45	73	1	22
	21	63	0	0	70	0	0	77	0	0

EXPLA-

EXPLANATION of the foregoing TABLE.

At the head of the table is set the thickness of the wall, in bricks and half bricks, from half a brick in thickness to six bricks.

In the first column is placed the number of the rods contained in any wall, from one quarter of a rod to twenty-one rods; and against those numbers, in the other columns, stands the quantity of brick-work, in rods, quarters, and feet which a wall contains of any of these thicknesses, at the head of the table.

EXAMPLE I.

If a wall upon the superficies contains twelve rods, and it be three bricks and a half in thickness, how many rods does the wall contain of the standard thickness of a brick and a half?

Find twelve rods in the first, and under three bricks, $\frac{1}{2}$, at the head of the table, in the angle of meeting, you will find 28 rods; which is the true quantity required at the standard thickness of a brick and half.

EXAMPLE II.

If a wall be four bricks and a half thick, and contains upon the superficies nineteen rods, how many rods of brick-work are contained in that wall, at the standard thickness?

Look for nineteen rods in the first column, and under 4 bricks $\frac{1}{2}$, you will find fifty-seven rods, the content required.

The same may be observed as to any other wall.

Discip. I think your directions so very plain, that I thoroughly understand the whole, and therefore shall not give you the trouble of answering any questions.

Philos. It gives me great pleasure to hear you understand my instructions so readily; indeed I have endeavoured to render them as plain and evident, as the nature

ture of the subject will admit. But before I conclude this chapter of bricklayer's work, it will be necessary to explain the method

Of measuring gable-ends.

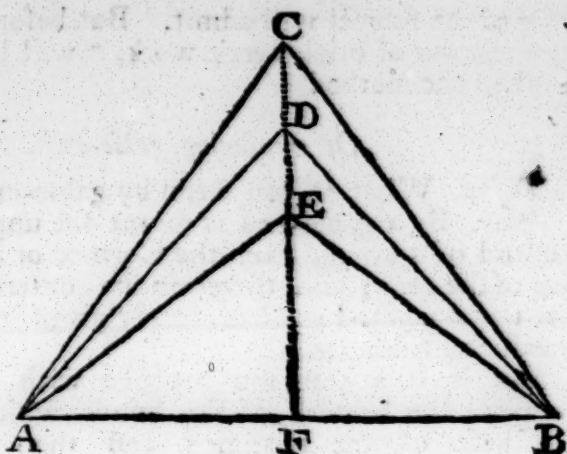
Discip. What do you mean by gable-ends?

Phil. By a gable-end is meant the upright triangular end of a house, from the Cornice or Eaves, to the top of the roof; and therefore the content is found by the same method as that already given in page 17, for measuring triangles.

But it is a common practice with workmen to measure the breadth of the house, which constitutes the base of the triangle; and then measure the length of the rafter, adding to it half its length; and the half of that sum will be the length of the perpendicular required. But this rule is true only when the roof is what they call *a true pitch*; that is, when the rafters are three quarters of the breadth of the building.

I would therefore always advise you to measure the length of the perpendicular; or, if that cannot be easily done, to square both the length of the rafter, and half the length of the base; subtract the lesser from the greater, and extract the square root out of the remainder, which will be the true length of the perpendicular required.

As the measuring of gable-ends is of some consequence in finding the contents of bricklayers work, and is often mistaken by workmen, the diagram before you



will explain the true method of proceeding, and point out the errors that often arise by following the common rule, with regard to the length of the perpendicular.

The triangle ADB, represents a gable-end of a true pitched roof, that is, when the length of the rafter $AD = DB$, is three fourths of the length of the base AB. In this case the length of the perpendicular DF, will be truly found by the common method: But if the roof be above true pitch; that is, if the rafter AC be more than three fourths of the base AB, the length of the perpendicular CF, cannot be found sufficiently near the truth by the common rule. For CF is considerably greater than DF; and consequently the whole space between ACB and ADB will be lost to the workman. On the contrary, if the roof be below the true pitch; that is, if the rafter $AE = EB$ be less than three fourths of the base AB, the perpendicular EF will be shorter than DF; and consequently the workman will be paid for the space between ADB, and AEB, above what he has really done. I would therefore advise you never to trust to the workmen's common rule, 'till you are assured by actual mensuration, that the roof is truly pitched; because you are then sure of being right, and because the true method is attended with very little trouble, more than the common rule.

These

These particulars being well considered, you will find no difficulty in measuring any kind of works performed by the bricklayer: I shall therefore finish this species of mensuration, with solving the following question.

What will the bricklayer's work for erecting the walls of a house, at 5l. 10s. the rod, amount to, supposing the walls to be of the following dimensions: The side-walls 28 feet 6 inches in length, the height of the roof from the ground 55 feet 6 inches, and the gable to rise 10 feet 6 inches, the roof being a true pitch: The side-walls from the bottom to 20 feet at 2 bricks and a half thick: 20 feet more at 2 bricks: 15 feet 6 inches more at 1 brick and a half thick; and the gable-end 1 brick thick?

SOLUTION.

FIRST DIMENSIONS.

$$\begin{array}{rcl}
 28.5 & \text{The length.} & \\
 \underline{20} & & \\
 570.0 & & \\
 \underline{5} & \text{Half bricks thick.} & \\
 3 \overline{) 2850 \cdot 950} & \text{Area in feet.} &
 \end{array}$$

$$\begin{array}{r}
 27 \\
 \hline
 15 \\
 15 \\
 \hline
 \cdot \cdot 0
 \end{array}$$

SECOND

THE WHOLE ART

SECOND DIMENSIONS.

$$\begin{array}{r}
 28.5 \\
 \underline{20} \\
 570.0 \\
 4 \\
 \hline
 3 \overline{) 228.01760} \quad \text{Area in feet.} \\
 \underline{21} \\
 \cdot 18 \\
 \underline{18} \\
 \cdot \cdot 0
 \end{array}$$

THIRD DIMENSIONS.

$$\begin{array}{r}
 28.5 \\
 \underline{15.5} \\
 1425 \\
 \underline{1425} \\
 285 \\
 \underline{441.75} \quad \text{Area in feet.}
 \end{array}$$

FOURTH DIMENSIONS, OR GABLE-END.

$$\begin{array}{r}
 28.5 \\
 \underline{5.3} \quad \text{Half perpendicular of the gable.} \\
 855 \\
 \underline{1425} \\
 151.05 \\
 \underline{2} \\
 3 \overline{) 302.101100.7} \quad \text{Area in feet.} \\
 \underline{3 \cdot \cdot} \\
 \cdot 021 \\
 \underline{21} \\
 \cdot \cdot
 \end{array}$$

950 First dimensions.

760 Second

441.75 Third

100.7

2252.45

Then as 272 feet, (a square rod)

F.

L.

is to 5.5 : : 2252.45 : 45.545

5.5

126225

1126225

272)12388.475

1088

1508

1360

1484

1360

1247

1088

1595

1360235

(45.545

20

10.900

12

10.800

4

3.200

Answ. 45l. 10s. 10d. $\frac{3}{4}$

CHAP. III.

Of measuring CARPENTERS WORK.*Discip.* What methods are made use of by carpenters in measuring their works?*Philo.* Carpenters, in measuring their works, have recourse to different methods; because the different parts of their employment render different methods of mensuration necessary. But at the same time you must remember, that these methods vary only in taking the dimensions; for when these are truly found, the contents will be had, from the common method of measuring geometrical figures.*Discip.*

Discip. This explanation has removed several difficulties which occurred to my mind. I was persuaded, that the areas were found by the same methods that are used for finding them in similar figures; and therefore required very little explanation; as I hope, I have sufficiently retained all the rules you have been kind enough to lay down for that purpose. But I shall now listen very attentively to your explanation of the methods taken by these practical artists.

Philo. Carpenters works, which are measurable, are flooring, partitioning, and roofing, all which are measured by the square of ten feet long, and ten broad, so that one square contains an hundred square feet.

OF FLOORING.

The dimensions of floors are taken within the walls; but those of joists and girders are measured by the length of the room, and also by the distance they are let into the walls.

EXAMPLE. I.

If a floor be 57 feet 3 inches long, and 28 feet 6 inches broad, how many squares of flooring are there in that room? To know this, multiply 57 feet 3 inches, by 28 feet 6 inches, and the product will be 1631; divide this by 100, which is done by cutting off two figures towards the right hand with a dash of the pen, and the remaining figures are the quotient, and the figures cut off are feet. Thus if you divide 1631 by 100, that is cut off 31 from the right hand, the quotient will be 16 squares, and the 31 cut off be 31 feet.

Operation by decimals, and also by feet and inches.

$$\begin{array}{r}
 57.25 \\
 28.5 \\
 \hline
 28625 \\
 45800 \\
 11450 \\
 \hline
 16|31.625
 \end{array}$$

$$\begin{array}{r}
 \text{F. I.} \\
 57.3 \\
 28.6 \\
 \hline
 456 \\
 114 \\
 28.76 \\
 7.00 \\
 \hline
 16|31.76
 \end{array}$$

Answ. 16 squares, 31 feet.

EXAMPLE II.

If a floor be 53 feet 6 inches long, and 47 feet 9 inches broad; how many squares are contained in that floor?

OPERATION.

$$\begin{array}{r}
 47.75 \\
 53.5 \\
 \hline
 23875 \\
 14325 \\
 23875 \\
 \hline
 25|54.625
 \end{array}$$

$$\begin{array}{r}
 \text{F. I.} \\
 53.6 \\
 47.9 \\
 \hline
 371 \\
 212 \\
 26.9 \\
 13.46 \\
 23.106 \\
 \hline
 25|55.00
 \end{array}$$

Answ. 25 squares, 55 feet

Of PARTITIONING.

The dimensions of partitions are taken in the same manner as those of floors, and therefore require no farther explanation.

EXAMPLE I.

If a partition between rooms be in length 82 feet 6 inches, and in height 12 feet 3 inches; how many squares are contained therein?

I

Operation

Operation by decimals, and also by feet and inches.

$$\begin{array}{r}
 12.25 \\
 82.5 \\
 \hline
 6125 \\
 2450 \\
 \hline
 9800
 \end{array}$$

10|10.625 Answ. 10 squares, 10 feet.

$$\begin{array}{r}
 \text{F. I.} \\
 82.6 \\
 12.3 \\
 \hline
 990.0 \\
 20.7.6 \\
 \hline
 10\ 10.7.6
 \end{array}$$

EXAMPLE II.

If a partition between rooms, be in length 19 feet 9 inches, and in breadth 11 feet 3 inches; how many squares are contained therein?

Operation by decimals.

$$\begin{array}{r}
 19.75 \\
 11.25 \\
 \hline
 9875 \\
 3950 \\
 1975 \\
 \hline
 1975 \\
 \hline
 2|22.1875
 \end{array}$$

Answer, 2 squares, 22 feet.

OF ROOFING.

It is a rule among workmen, that the flat of any house, and half the flat thereof taken within the walls, is equal to the measure of the roof of the same house; but this is when the roof is true pitched; for if the roof be more flat or steep than the true pitch, it will measure more or less accordingly; multiply therefore the length and breadth together, to that product add half the sum, and the total being divided by 100, will give the answer in squares.

EXAMPLE.

EXAMPLE.

If a house within the walls be 44 feet 6 inches long, and 18 feet 3 inches broad, how many squares of roofing will cover that house?

$$\begin{array}{r}
 18.25 \\
 41.5 \\
 \hline
 9.125 \\
 7300 \\
 \hline
 7300 \\
 \hline
 \text{Flat } 812.125 \\
 \text{Half } 406 \\
 \hline
 \end{array}$$

12|18 Answ. 12 sq. 18 feet.

$$\begin{array}{r}
 \text{F. I.} \\
 44.6 \\
 18.3 \\
 \hline
 352 \\
 44 \\
 11.1.6 \\
 9.0.0 \\
 \hline
 \text{Flat } 812.11.6 \\
 \text{Half } 406 \\
 \hline
 12|18
 \end{array}$$

In the measuring-work of carpenters, there are two things which should be attended to:

First, in the measuring of flooring, after the whole floor has been measured, there are to be deducted out of it the well-holes for the stairs and chimnies; and in partitioning, for the doors, windows, &c. except, by agreement, they are to be included.

Secondly. In measuring of roofing, seldom any deductions are made for the holes for the chimney shafts, or the vacancies for the lanthorn-lights and sky-lights, as they are more trouble to the workman, than the stuff that would cover them is worth.

CHAP. IV.

OF MEASURING GLAZIERS-WORK.

Glaziers generally measure their work to a quarter of an inch, and never make any allowances for round or oval windows, but always measure them to the greatest length; for there is more trouble in cutting the glass to those shapes, than the value of the glass omitted.

When the dimensions are thus accurately taken, the content in feet, inches, &c. is easily found by the methods

thods already laid down for measuring superficies, and which need not be repeated here. But as these tradesmen give particular appellations to the various dimensions of quarries used in glazing windows; it will be necessary to add an explanation of these terms, and then shew how they are measured, and the content found in feet, inches, and parts.

The range, which is a perpendicular let fall from one of the obtuse angles to the opposite side, when the length is the longest diagonal, from one acute angle to the other, and the breadth is the shortest diagonal, which is drawn between the two obtuse angles; as for the sides, and area of the quarry, these are well known to all.

Glaziers generally consider twelve sorts of quarries, from whence arise divers propositions of infinite use to Glaziers; as

First. To find any of the dimensions, as range, side, length, and breadth, of any sort of quarries.

Secondly. To find the area of any sort of quarries.

Thirdly. Having any of the dimensions given, viz. range, side, breadth or length, to find the name or denomination of the size, viz. whether 8s. 10s. 12s. &c.

Fourthly. Having the area of a quarry given, to find of what sort or size it is.

Fifthly. To find whether a window be glazed with those, they call square quarries or long ones; for it is to be observed, that there are six sizes of square quarries, and six sizes of long quarries, which make twelve sorts in all.

Glazier's work is measured by the foot square; so that the length and breadth of a pane of glass in feet, being multiplied into each other, produceth the content.

It should be remarked that Glaziers usually take their dimensions very accurately; and in multiplying feet inches, and parts, the inch is divided into 12 parts, as the foot is, and each part is also divided into 12.

EXAMPLE I.

If a pane of glass be four feet eight inches, and three quarters long; and one foot, four inches, and one

OF MEASURING.

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one quarter broad, how many feet of glass does it contain?

The decimal of $\left\{ \begin{array}{l} 8 \text{ Inches} \frac{3}{4} \\ 4 \text{ Inches} \frac{1}{4} \end{array} \right\}$ is $\left\{ \begin{array}{l} .729 \\ .354 \end{array} \right\}$

F. I. P.				
4	8	9		
1	4	3		
4	8	9		
1	6	11	0	
	1	2	2	3
6	4	8	2	3

$$\begin{array}{r} 4.729 \\ 1.354 \\ \hline 18916 \\ 23645 \\ 14187 \\ 4729 \\ \hline 6.403066 \end{array}$$

Facit 6 feet 4 inches.

EXAMPLE II.

If there be eight panes of glass, each four feet, seven inches, three quarters long, and one foot five inches one quarter broad, how many feet of glass are contained in the said eight panes?

The decimal of $\left\{ \begin{array}{l} 7 \text{ Inches} \frac{3}{4} \\ 5 \text{ Inches} \frac{1}{4} \end{array} \right\}$ is $\left\{ \begin{array}{l} .646 \\ .437 \end{array} \right\}$

F. I. P.				
4	7	9		
1	5	3		
4	7	9		
1	11	2	9	
	1	1	11	3
6	8	1	8	3
				8
53	5	1	6	0

4.646
I .45
32522
13938
18584
4646
6.676302
8
53.410416

Facit 53 feet 5 inches.

EXAMPLE III.

If there are sixteen panes of glass, each four feet, five inches, and a half long, and one foot four inches and three

three quarters broad, how many feet of glass are contained in them?

F.	I.	P.		
4	:	5	:	6
1	:	4	:	9
4	:	5	:	6
1	:	5	:	10
		3	:	4
6	:	2	:	8
			:	1
			:	6
				4
24	:	10	:	8
			:	6
				4
99	:	6	:	10
			:	0
			:	0

4	458
1	395
	22290
	40122
	13374
	4458
	6.218910
	4
	24.875640
	4
	99.502560

Facit 99 feet 6 inches.

It may be observed, that instead of multiplying by 16, I have multiplied by 4 twice, because four times 4 is 16.

When windows have half rounds at the top, they are to be measured at their full height, as if they were square. For the same reason, round or oval windows are measured at the full length and breadth of their diameters.

In like manner are crotchet-windows in stone-work measured by their full squares; and for this plain reason, that the trouble of taking their dimensions to work by, the waste of glass in working, and the time spent in setting them up, is far more than the value of the glass.

C H A P. V.

OF MEASURING JOINERS-WORK.

Joiners measure their work by the yard square; but they take their dimensions differently from others. In taking the height of a room, about which there is either cornice, swelling pannels, or mouldings, they begin at the top with a string, and girt over all the mouldings, which makes the room measure much higher than it really

OF MEASURING.

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really is ; but in measuring about the room, they take it only as it is upon the floor.

RULE.

Multiply the compass by the height, and the product divided by 9 will give the answer.

EXAMPLE I.

If a room of wainscot being girt downwards over the mouldings be 15 feet 9 inches high, and 126 feet 3 inches in compass, how many yards are contained in that room ?

F.	I.
126	: 3
15	: 9
630	
126	
63	: 1 : 6
31	: 6 : 9
3	: 9 : 0
9)1988	: 5 : 3

126	: 25
15	: 75
63125	
88375	
63125	
12625	
9)1988.4375	
220	: 8

Answ. 220 Y : 8 F.

EXAMPLE II.

If a room of wainscot be 16 feet 3 inches high, girt over the mouldings, and the compass of the room is 137 feet 6 inches, how many yards does it contain ?

$$\begin{array}{r}
 \text{F.} \quad \text{I.} \\
 137 : 6 \\
 \hline
 16 : 3 \\
 \hline
 830 \\
 137 \\
 \hline
 34 : 4 : 6 \\
 \hline
 9)2234 : 4 : 6 \\
 \hline
 248 : 2 : 0
 \end{array}$$

$$\begin{array}{r}
 137.5 \\
 162.5 \\
 \hline
 6875 \\
 2750 \\
 8250 \\
 1375 \\
 \hline
 9)2234.375 \\
 \hline
 248.2
 \end{array}$$

Answ. 248 yards. 2 feet.

In measuring doors, window-shutters, and all such work, as is wrought on both sides, joiners are paid for work and half work; therefore in measuring all such work, the content must first be found as before, and half that content must be added to it, which will give the content at work and half.

EXAMPLE III.

If the window-shutters be 69 feet 9 inches broad, and 6 feet 3 inches high, what is their content at work and half?

$$\begin{array}{r}
 \text{F.} \quad \text{I.} \\
 69 : 9 \\
 \hline
 6 : 3 \\
 \hline
 418 : 6 \\
 \hline
 17 : 5 : 3 \\
 \hline
 435 : 11 : 3 \\
 \hline
 217 : 11 : 7 \\
 \hline
 9)653 : 10 : 10 \\
 \hline
 72 : 5
 \end{array}$$

$$\begin{array}{r}
 69.75 \\
 6.25 \\
 \hline
 34875 \\
 13950 \\
 41850 \\
 \hline
 435.9375 \\
 217.9687 \\
 \hline
 9)653.9062 \\
 \hline
 72.5
 \end{array}$$

Answ. 72 yards, 5 feet.

C H A P. VI.

OF MEASURING PAINTERS-WORK.

The method of taking the dimensions here, is the same with that of joiners, by girting over the mouldings, &c. in taking the height. The casting up, after the dimensions have been taken and reduced into yards, is likewise the same with that of joiners work; but the painter never reckons work and half; but reckons his work once, twice or thrice coloured over.

It should however be observed, that window-lights, window-bars, casements, and such like things, are done at so much per piece.

EXAMPLE.

If a room be painted, whose height, being girt over the mouldings, is 16 feet 6 inches, and the compass of the room be 97 feet 9 inches, how many yards are there in that room?

$$\begin{array}{r}
 \text{F. I.} \\
 97 : 9 \\
 16 : 6 \\
 \hline
 584 \\
 98 \\
 48 : 10 : 6 \\
 \hline
 9)1612 : 10 : 6
 \end{array}$$

$$\begin{array}{r}
 97.75 \\
 16.5 \\
 \hline
 48875 \\
 58650 \\
 9775 \\
 \hline
 9)1612.875
 \end{array}$$

Answer 179 yards, 1 foot.

C H A P. VII.

OF MEASURING PLASTERERS-WORK.

Plasterers works are of two kinds:

First, Lathed and plastered work, which they call ceiling.

Secondly, Rendered work, which is of two sorts, viz. upon brick-walls or between quarters, and in the partitions between rooms; all which are measured

THE WHOLE ART

by the yard square, or square of 3 feet, which is 9 feet.

OF CIELING.

If a ceiling be 59 feet, 9 inches long, and 24 feet, 6 inches broad, how many yards are contained in that cieling?

EXAMPLE.

F. I.	
59 : 9	59.75
24 : 6	<u>24.5</u>
236	29875
118	23900
18 : 00 : 0	<u>11950</u>
29 : 10 : 6	9)1463.875
9)1463 : 10 : 6	Answ. 162 yards, 5 feet.

OF RENDERING.

If the partitions between rooms be 141 feet 6 inches about, and 11 feet 3 inches high, how many yards do these partitions contain?

EXAMPLE.

F. I.	
141 : 6	141.5
11 : 3	<u>11.25</u>
1556 : 6	7075
35 : 4 : 6	2830
9)1591 : 10 : 6	1415
Answ. 176 : 7 : 0	1415
	9)1591.875
	176.87

There are three things here which should be particularly observed,

First, If there are any doors, windows, or the like, in the partitioning, deductions must be made for them.

Secondly, When rendering upon brick-walls is measured, no deductions are to be made; but when rendering is measured between quarters, one fifth part may very well be deducted for the quarters, braces, and interstices.

Thirdly, Whiting and colouring are both measured by the yard, as ceiling and rendering were; and likewise in whiting and colouring, one fourth, or at least, one fifth part is to be added, as one fifth part is deducted, in rendering between quarters.

CHAP. VIII.

OF PLUMBERS WORK.

Discip. **W**HAT method can be taken to measure Plumber's Work?

Tyro. No direct method can be taken; because it is impossible, by the arts now in use, to cast lead of an equal thickness throughout. And accordingly, all Plumber's Work is undertaken by the pound or hundred weight.

Many experiments have been tried to reduce this to some certainty, by measuring two parcels of the same dimensions, and cast at the same time; but they have all proved abortive; one of the parcels, though exactly of the same dimensions, has always been found to be much heavier than the other. Besides, some work requires a much greater quantity of solder than others, and consequently must be dearer, the solder being of much more value than lead.

Discip. I am greatly obliged to you for this information; as I might otherwise have attempted, from measuring the surface, and the thickness in some particular part, to calculate the value of the lead covering any building; and by that means have exposed myself to ridicule.

B O O K III.

Of the MENSURATION of SOLIDS:

Discip. **W**HAT am I to understand by the word Solid?

Philo. By a solid is meant a figure contained under three dimensions, length, breadth, and depth or thickness; that is, the thickness of the body is here considered, which, in measuring superficies is disregarded.

To make this the more easy to be comprehended, let us suppose that a board is four inches thick; it is evident, that this board may be cut into four, the surface of each of which will measure the same as before the board was cut. But it would be very unreasonable to suppose that a board of four inches thick should be sold at the same price per foot, as another which is only one. It is therefore often necessary to consider the depth or thickness of bodies required to be measured; and this is what we mean by the mensuration of solids; or finding the content of bodies that have three dimensions.

Solids are of various kinds; and therefore require various methods to find their contents.

1. If the ends and sides are equal squares, the solid is called a cube.
2. If the bases or ends are both equal, and both at right angles; the solid is called a parallelopiped.
3. It is called a cylinder when the ends are equal circles.
4. If the solid be formed on a triangular, square, or polygonal base, having its sides plain triangles, whose vertices all meet in the same point, it is called a pyramid; and is denominated from the number of sides of which its basis consists; as triangular, square, &c.
5. If the bases, or ends, are equal, parallel, and similar right lined plain figures, and the sides parallelograms, it is called a prism.
6. If the base be a circle, and the top or apex a point, it is called a cone.

7. If

7. If the solid be terminated by a convex surface, whereof every point is equally distant from a certain point within the solid, it is called a sphere.

8. If the solid resemble a sphere, but one of its diameters be longer than the other; that is, if the superficies be in the form of an oval, it is called a Spheroid.

These are the principal figures, whose solid content is required to be found, and therefore I shall first, my dear Discipulus, explain the methods generally used for that purpose, before I proceed to lay down the rules necessary to be observed in measuring frustums, &c.

Of a CUBE.

A Cube is a regular or solid body, consisting of six square or equal faces or sides, and its angles all right and therefore equal.

It is supposed to be generated by the motion of a square plane along a line equal to one of its sides, and at right angles thereto; whence it follows, that the planes of all sections parallel to the base, are squares equal thereto, and consequently equal to one another.

A cube being a square solid, comprehended under six geometrical squares, in the form of a dye, the solid content of this figure is easily found by the following

R U L E.

Multiply the side of the cube into itself, and that product again by the side; the last product will be the solidity, or solid content of the cube.

E X A M P L E.

Suppose the side of a cube be seventeen inches and a half, what is the solid content?

OPERATION.

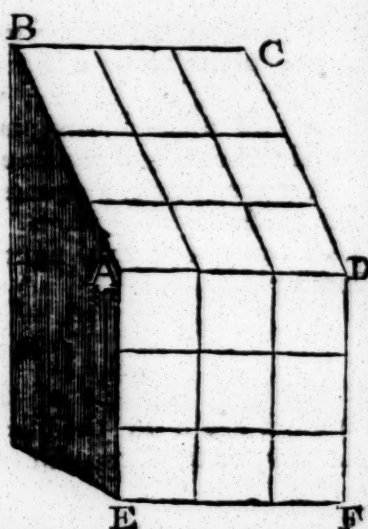
OPERATION.

$$\begin{array}{r}
 17.5 \\
 \underline{17.5} \\
 875 \\
 1225 \\
 \underline{175} \\
 306.25 \\
 \underline{17.5} \\
 153125 \\
 214375 \\
 \underline{30625}
 \end{array}$$

5359.375 the solid content.

Suppose ABCDFE a cubical piece of stone, each side thereof being seventeen inches and a half, multiply 17.5 by 17.5, and the product is 306.25; which being multiplied by 17.5, the last product will be 5359 solid inches, and 375 parts.

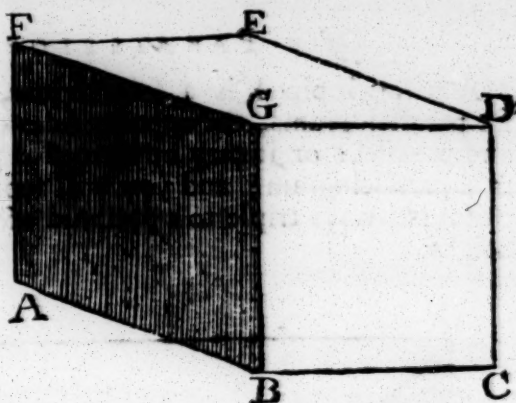
To reduce the solid inches to feet, divide by 1728, because there are so many cubical inches in a foot, and the solid feet in the cube will be 3, and 175 cubical inches remain.



DEMON-

DEMONSTRATION.

If the square FEDG be conceived to move down the plane AFGB always remaining parallel to itself, there will be generated by that motion a solid, having six planes; the two opposite of



which will be equal and parallel to each other, whence it is called a parallelopipedon, or square prism.

If the plane AFGF be a square equal to the generating plane FEDG, then will the generated solid be a cube.

From hence such solids may be conceived to be constituted of an infinite series of equal squares, each equal to the square FEDG, and AF or BG will be the number of terms.

If the area, therefore, of FEDG be multiplied into the number of terms AF, the product is the sum of all the series, and consequently the solidity of the parallelopipedon, or cube.

Or if the base FEDG, being divided into little square areas, be multiplied into the height AF divided into an equal number, you may conceive as many little squares to be generated in the whole solid, as is the number of the little areas of the base, multiplied by the number of divisions on the side AF.

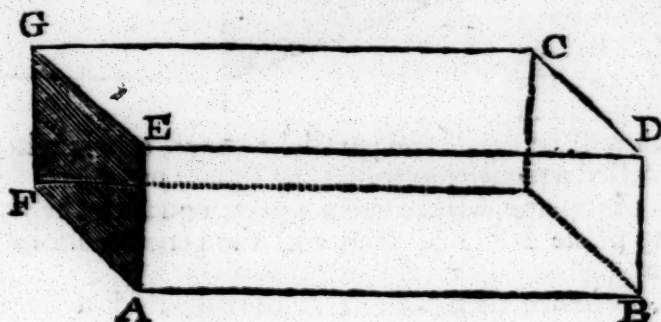
Thus if the side of the base FE be 10, that multiplied by itself is 100, which is the area of the square base FEDG; then if AF be likewise 10, multiply 100 by 10, and the product will be 1000, and so many little cubes may this solid be cut into.

From this demonstration it plainly appears, that if you multiply the area of the base of any parallelopiped into

into its height, that product will be the solid content of such a solid.

Of a PARALLELOPIPED.

A parallelopiped is a solid figure, contained under six parallelograms, the opposite sides of which are equal and parallel; or it may be termed a prism, whose base is a parallelogram; and you will soon be shewn that a prism is always triple to a pyramid of the same base and height.



Let $ABDCGF$ be a parallelopipedon or a square prism, representing a square piece of timber or stone, each side of its square base $AEGF$, being 21 inches, and its length AB , 180 inches or 15 feet.

First then multiply 21 by 21, the product is 141, the area of the base in inches; which multiplied by 180, the length, the product is 79380, the solid content in inches.

Divide the last product by 1728, and the quotient is 45.9, that is, 45 solid feet, and 9 tenths of a foot.

Or thus, multiply 441 by 15 feet, and the product is 6615; divide this by 144 and the quotient is 45.9, the same as before.

Or thus, by multiplying feet and inches, multiply 1 foot 9 inches, by 1 foot 9 inches, and the product is 3 feet 0 inches, 9 parts; this multiplied again by 15 feet, gives 45 feet, 11 inches, 3 parts; that is 45 feet and $\frac{11}{12}$ of a foot, and $\frac{3}{4}$ of $\frac{1}{12}$.

See

See the Work.

F. I.			
21	44 ¹	1 9	1728)79380(45.9
<u>21</u>	<u>15</u>	<u>1 9</u>	<u>6912</u>
21	2205	1 9	10260
<u>42</u>	<u>44¹</u>	<u>1 3 9</u>	<u>8640</u>
44 ¹	144)6615(45.9	3 0 9	16200
<u>180</u>	<u>855</u>	<u>15</u>	<u>15552</u>
35280	<u>1350</u>	45 11 3	<u>648</u>
<u>44¹</u>	<u>54</u>		
79380			

If the base of the solid be not an exact square, or not in the form of a rectangled parallelogram, the method of measuring is much the same; for first you must find the area of the base by multiplying the breadth by the depth, and then multiply that area by the length of the piece as before: thus,

If a piece of timber be 25 inches broad, 9 inches deep, and 25 feet long, how many solid feet are contained therein?

F. I.		F. I.	
25	144)5625(39	2	1
<u>9</u>	<u>432</u>	<u>0 9</u>	
225	1305	1 6 9	
<u>25</u>	<u>1296</u>	<u>25</u>	
1125	09	25 : 0 0	
<u>450</u>		12 : 6 0	
5625	Answ. 39 feet.	1 : 0 6	
		<u>0 : 6 3</u>	
		39 : 0 9	

Of a CYLINDER.

Discip. What species of solid is a cylinder?

Philo. A cylinder is a round solid, having its bases circular, equal, and parallel, in form of a rolling stone, and the solid content is found by the following

RULE.

R U L E.

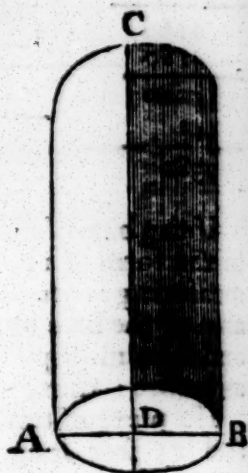
Multiply the area of the base by the length, and the product is the solid content.

E X A M P L E.

Let ABC be a cylinder, whose diameter AB is 21.5 inches, and the length CD 16 feet, what is the solid content?

First, square the diameter 21.5, and it makes 462.25, which multiplied by .7854. (See page 28) the product will be 363.05115.

Then multiply this by 16, and the product will be 5808.8164. Divide this last product by 144, and the quotient will be 40.34 feet, the solid content.

*To find the superficial Content.*

First, find the circumference of the base 67.54; which being multiplied by 16, the product is 1080.64; which divided by 12, the quotient will be 90.05 feet, the curved surface; to which add 5.04 feet, the sum of the bases, and the sum will be 95.09 feet, the whole superficial content. See the work.

$$\begin{array}{r}
 67.54 \\
 \hline
 16 \\
 \hline
 40524 \\
 6754 \\
 \hline
 12 \overline{) 1080.64} \\
 \underline{90.05} \\
 90.09
 \end{array}
 \quad
 \begin{array}{l}
 90.05 \\
 5.04 \\
 \hline
 95.09
 \end{array}
 \left. \vphantom{\begin{array}{l} 90.05 \\ 5.04 \end{array}} \right\} \text{add}$$

$$\begin{array}{r}
 363.05 \\
 \hline
 2 \\
 \hline
 144 \overline{) 726.10} (5.04 \\
 \underline{720} \\
 610 \\
 \underline{576} \\
 34
 \end{array}$$

DEMONSTRATION.

The solid content of every cylinder is found by multiplying the area of its base into the height, as above; for every right cylinder is only a round prism, being constituted of an infinite series of equal circles; that of its base or end being one of the terms; and its height CD , in the figure, is the number of all the terms.

Therefore the area of its base AB being multiplied into CD , will be its solidity.

Of a PYRAMID:

Discip. What do you mean by a Pyramid?

Philo. A Pyramid is a solid standing on triangular, square, or polygonal base; its sides are plane triangles, and the several tops meet together in one point. Most steeples are pyramids: the properties of a pyramid are,

1. A triangular pyramid is the third part of a prism standing on the same base, and of the same altitude.

2. Hence as every multangular may be divided into a triangular body, every pyramid is the third part of a prism standing on the same base, and of the same altitude.

3. If

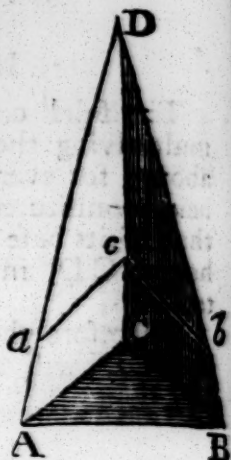
3. If a pyramid be cut by a plane abc , parallel to its base ABC the former plane or base will be similar to the latter.

4. All pyramids, prisms, cylinders, &c. are in a ratio compounded of their bases and altitudes: the bases therefore being equal, they are in proportion to their altitudes; and the altitudes being equal, they are in proportion to their bases.

5. Pyramids, prisms, cylinders, cones, and other similar bodies, are in a triplicate ratio of their homologous sides.

6. Equal pyramids, &c. reciprocate their bases and altitudes, that is, the altitude of the one is to that of the other, as the base of the one to that of the other. A sphere is equal to a pyramid, whose base is equal in the surface, and its height to the radius of the sphere.

7. The solidity of a pyramid is one third of the perpendicular altitude, multiplied by the base.



To measure the surface and solidity of a Pyramid.

Find the solidity of a prism that has the same base with the given pyramid, and divide this by three; the quotient will be the solidity of the pyramid.

EXAMPLE.

Suppose the solidity of the prism be found 6791031, the solidity of the pyramid will be thus found 2263677.

The external surface of a right pyramid standing on a regular polygonian base, is equal to the altitude of one of the triangles which compose it, multiplied by the whole circumference of the base of the pyramid.

To describe a Pyramid on a Plane.

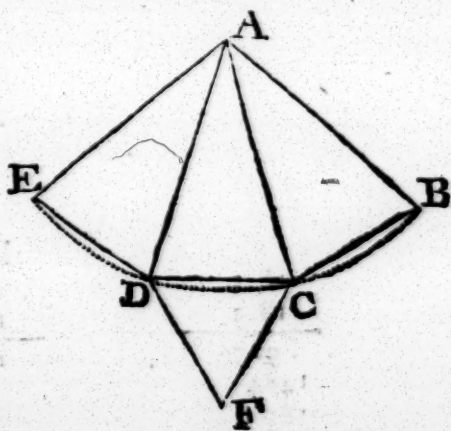
First, draw the triangle ABC for the base, if the pyramid required be triangular, so as that the side AB supposed to be turned behind, be not expressed. Secondly, on AC and CB, construct the triangles ADC, and CDB, meeting in any assumed or determined point, for example D; and draw AD, CD, and BD, then will ABCD be a triangular pyramid.

*To construct a Pyramid of Paste-Board, &c.*

Suppose it were required to construct a triangular pyramid.

1. With the radius AB, describe an arch BE, as in the figure, and there-to apply three equal chords BC, CD and DE.

2. On DC construct an equilateral triangle DFC, and draw the right lines AD and AC, this piece of paste-board, &c. being cut out from the centre of the figure, what remains within will turn up into a pyramid.



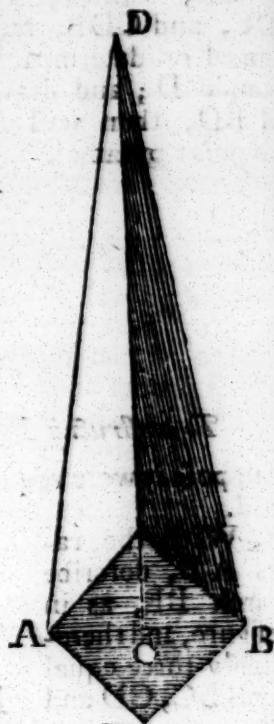
To find the solid Content of a Pyramid.

RULE.

Multiply the area of the base by a third part of the altitude, and the product will be the solid content of the pyramid.

EXAMPLE.

Let ABCDE be a square pyramid, each side of the base being 18.5 inches, and the perpendicular height CD 15 feet: multiply 18.5 by 18.5, and the product is 342.25 the area of the base in inches, which multiply by 5, a third part of the height, and the product of that will be 1711.25, this being divided by 144, the quotient will be 11.88 feet.



18.5
18.5
925
1484
185
342.25
5

Area of base 2 : 4 : 6.3
144) 1711.25 (11.88 content

F.	I.	P.
1	6	6
1	6	6
1	6	6
	9	3
		9.3
2	4	6.3
11	10	7.3

To find the superficial Content.

Multiply the slanting height, or perpendicular of one of the triangles, by half the periphery of the base 37, and the product will be 6668.88, which being divided by 144, the quotient is 46.31 feet, the superficial content of all except the base; then to that add 2.38 feet the base, and it makes 48.69 feet, the whole superficial content. See the work.

$ \begin{array}{r} 188.24 \\ \underline{37} \\ 126168 \\ 54072 \\ \hline 144 \overline{) 6668.88} \quad (46.31 \\ \underline{670} \quad 2.38 \\ 908 \quad 48.69 \\ \underline{864} \\ 448 \\ \underline{432} \\ 168 \\ \underline{144} \\ 24 \end{array} $	$ \begin{array}{r} 144 \overline{) 342.25} (2.38 \text{ nearly} \\ \underline{288} \\ 542 \\ \underline{432} \\ 1105 \\ \underline{1152} \end{array} $
---	---

EXAMPLE.

. EXAMPLE.

Let ABCDEGHI, be a pyramid, whose base is a heptagon, each side thereof being 15 inches, the perpendicular of the heptagon 15.58 inches, and the perpendicular height of the pyramid HI 13.5 feet; to find the solid and superficial content.

R U L E.

Multiply 15.58, the perpendicular, by 52.5, half the sum of the sides of the heptagon, and the product will be 817.95, which multiplied by 4.5, that is, $\frac{1}{3}$ of the height, and the product will be 3680.775.

Then divide this last product by 144, and the quotient will be 25.56 feet, the content.



15.58 the heptagon's perpendicular.

52.5 the half-sum of the sides.

$$\begin{array}{r} 7790 \\ 3116 \\ \hline 7790 \end{array}$$

$$\begin{array}{r} 817.950 \end{array}$$

4.5 a third part of the height.

$$\begin{array}{r} 4089750 \\ 3271800 \\ \hline \end{array}$$

144)3680.7750(25.56 solid feet:

$$\begin{array}{r} 288 \\ \hline 800 \\ 720 \\ \hline 807 \\ 720 \\ \hline 877 \\ 864 \\ \hline 13 \end{array}$$

To find the superficial Content.

Multiply the height taken from the middle of one of the sides of the base 162.75 inches, by the half-sum of the sides 52.5 inches, and the product will be 8544.375; which divided by 144, the quotient will be 59.335 feet, the content of the upper part.

$ \begin{array}{r} 162.75 \\ \underline{52.5} \\ 81375 \\ 32550 \\ 8 \ 375 \\ \hline 144)8544.375(59.335 \text{ Feet} \\ \underline{720} \qquad \underline{5.68} \text{ Base add} \\ 1344 \qquad 65.015 \text{ the whole content.} \\ \underline{1296} \\ \cdot 483 \\ \underline{432} \\ \cdot 517 \\ \underline{432} \\ \cdot 855 \\ \underline{720} \\ \underline{135} \end{array} $	$ \begin{array}{r} 144)817.95(5.68 \\ \underline{720} \\ 979 \\ \underline{864} \\ 1154 \\ \underline{1152} \\ 3 \end{array} $
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Of a PRISM.

Discip. What am I to understand by a Prism?

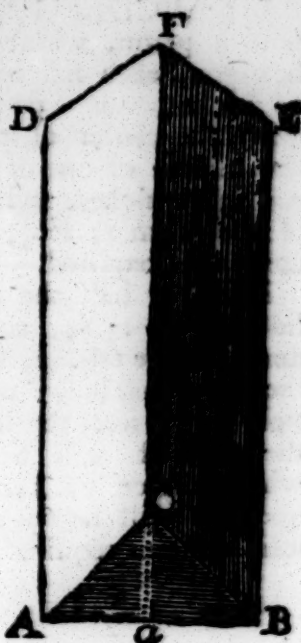
Phil. A Prism is a solid contained under several planes, and having its bases alike, equal, and parallel. The solid content of a prism, whether triangular or multangular, is found by multiplying the area of the base into the length or height, and the product will be the solid content.

EXAMPLE.

EXAMPLE.

Let $ABCDEF$ be a triangular prism, each side of the base being 15.6 inches, the perpendicular of it CD 13.51 inches, and the length of the solid 19.5 feet. This and all other questions of the same kind may be easily solved by the following rule.

Multiply the perpendicular of the triangle 13.51. by half the base 7.8, and the product will be 105.378, the area of the base; which multiplied by the length 19.5, the product will be 2054.871; which divided by 144, the quotient will be 14.27 feet nearly, the solid content required.



$$\begin{array}{r}
 13.51 \\
 \times 7.8 \\
 \hline
 10808 \\
 9457 \\
 \hline
 105.378 \\
 \times 19.5 \\
 \hline
 526890 \\
 948402 \\
 \hline
 105378 \\
 \hline
 2054.8710
 \end{array}$$

$$\begin{array}{r}
 144 \overline{) 2054.8710} \\
 \underline{144} \\
 614 \\
 \underline{576} \\
 .388 \\
 \underline{288} \\
 1007 \\
 \underline{1008} \\
 10
 \end{array}$$

EXAMPLE.

Let ABCDEFGHIK represent a prism, whose base is a hexagon, each side of which is 16 inches, the perpendicular from the center of the base to the middle of one of the sides, *ab*, 13.84 inches, and the length of the prism 15 feet; the solid content is required.

Multiply half the sum of the sides 48 by 13.84, and the product will be 664.32, the area of the hexagonal base; which multiply by 15 feet, the length, and the product will be 9964.80, which divide by 144, and the quotient will be 69.2 feet, the solid content required.



$$\begin{array}{r}
 13.84 \\
 \times 48 \\
 \hline
 11072 \\
 5536 \\
 \hline
 664.32 \text{ Area of the base.} \\
 \times 15 \\
 \hline
 332160 \\
 66432 \\
 \hline
 9964.80
 \end{array}$$

$$\begin{array}{r}
 144 \overline{) 9964.80} (69.2 \\
 \underline{864} \\
 1324 \\
 \underline{1296} \\
 288 \\
 \underline{288} \\
 000
 \end{array}$$

To find the superficial content of any of these solids, you must take the girt of the piece, and multiply by the length, and to that product add the two areas of the bases, and the sum will be the whole superficial content.

For

For example, in the hexagonal prism; the sum of the sides being 96, and the length 15 feet, that is 180 inches, multiply these into each other, and the product will be 17280 square inches, to which add twice 664.32 the areas of the two bases; and the sum is 18608.64, the area of the whole; which is equal to 129.22 feet.

$$\begin{array}{r}
 180 \\
 96 \\
 \hline
 1080 \\
 1620 \\
 \hline
 17280 \\
 664.32 \\
 664.32 \\
 \hline
 18608.64
 \end{array}$$

$$\begin{array}{r}
 144 \overline{) 18608.64} \quad 129.22 \\
 \underline{144} \\
 420 \\
 \underline{288} \\
 1328 \\
 \underline{1296} \\
 326 \\
 \underline{288} \\
 384 \\
 \underline{288} \\
 96
 \end{array}$$

The superficial content of the whole solid is 129.22 feet.

Of a CONE.

Discip. How do you define a Cone?

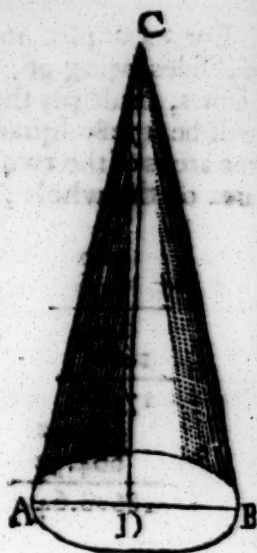
Phil. A Cone is a solid, having a circular base, and growing smaller and smaller, 'till it ends in a point, which is called the vertex, and may be nearly represented by a sugar-loaf. Its solidity will be found by the following

RULE.

Multiply the area of the base by a third part of the perpendicular height, and the product is the solid content.

Let ABC be a cone, the diameter of whose base AB is 26.5 inches; and the height of the cone DC is 16.5 feet.

First square the diameter 26.5, and it will make 702.25, which multiplied by .7854, the product will be 551.54715; this being multiplied by 5.5 the product will be 3033.509325, which being divided by 144, the quotient will be nearly 21.066, the solid content of the cone.



$ \begin{array}{r} 144 \overline{) 3033.509, \text{ \&c.}} \quad (21.066 \text{ feet} \\ \underline{288} \\ 153 \\ \underline{144} \\ 950 \\ \underline{864} \\ 869 \\ \underline{864} \\ \end{array} $	$ \begin{array}{r} 26.5 \text{ the diameter} \\ \underline{26.5} \\ 1325 \\ 1590 \\ \underline{530} \\ 702.25 \text{ Square} \\ \underline{.7854} \\ 280900 \\ 351125 \\ 561800 \\ \underline{491575} \\ 551.54715 \text{ Area of the base} \\ \underline{5.5 \frac{1}{2} \text{ Part of the height}} \\ 275773575 \\ \underline{275773575} \\ 3033.509325 \end{array} $
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To find the superficial Content.

Multiply half the circumference 41.626 by the slant height AC 198.46, and the product will be 8261.09596; which

OF MEASURING.

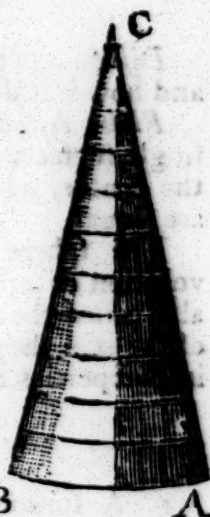
147

which being divided by 144, the quotient is nearly 57.37, the curve surface, to which add the base, and the sum will be 61.2 feet, the superficial content.

$ \begin{array}{r} 41.626 \\ \underline{198.46} \\ 249756 \\ 166504 \\ 333008 \\ 374634 \\ \underline{41626} \\ 8261.09596 \end{array} $	$ \begin{array}{r} 144 \overline{)8261.09596} \\ \underline{1061} \\ 530 \\ \underline{989} \\ 144 \overline{)551.54} \\ \underline{1195} \\ 434 \\ \underline{2} \end{array} $	57.37 Feet nearly. 3.83 Base. 61.20 whole cont.
---	---	--

DEMONSTRATION.

Every cone is the third part of a cylinder of equal base and altitude. The truth of this may easily be conceived if we only consider that a cone is but a round pyramid, and therefore it must needs have the same ratio to its circumscribing cylinder, as the square pyramid hath to its circumscribing parallelopipedon, that is 1 to 3. However to make it clearer, let it be considered, that every right cone is constituted of an infinite series of circles whose diameters continually increase in arithmetical progression, beginning at the vertex or point C, the area of its base AB being the greatest term, and its perpendicular height DC, the number of all terms, therefore the area of the circle of the base multiplied by a third part of the altitude DC, will be the sum of all the series, equal to the solidity of the cone.

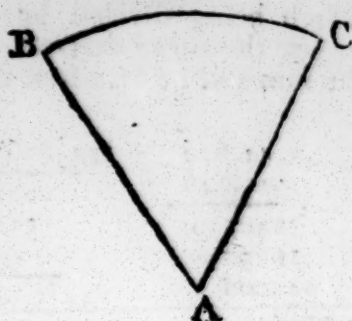


The curve superficies of every right cone is equal to half the rectangle of the circumference of its base into the length of its sides.

K 4

For

For the curve surface of every right cone is equal to the sector of a circle, whose arch BC is equal to the periphery of the base of the cone, and the radius AB equal to the slant side of the cone.



This will evidently appear, if you cut a piece of paper in the form of a sector of a circle as ABC, and bend both the sides AB and AC together till they meet, and you will find it to form a right cone.

Of a SPHERE.

Discip. What species of solid do you call a sphere; and how is it distinguished from other solids?

Philo. A sphere is a solid body contained under one single surface, and having a point in the middle, called the center, whence all the lines drawn to the surface are equal.

The sphere is supposed to be generated by the revolution of a semi-circle about its diameter, which is also called the *axis of the sphere*, and the extreme points of the axis, the *poles of the sphere*. And hence the following properties of the sphere will be easily conceived.

Properties of the Sphere.

1. A sphere is equal to a cone, whose base is equal to the surface, and its height to the radius of the sphere. Hence a sphere being considered as such a cone, its cube or solid content is found like that of a cone.

2. A sphere is to a cylinder, standing on an equal basis, and of the same height as 2 to 3. Hence also may the cube or content be found.

3. The

3. The cube of the diameter of a sphere, is to the solid content of the sphere, nearly as 300 to 157 : and thus also may the content of the sphere be measured.

4. The surface of a sphere is quadruple to that of a sphere described with the radius of a sphere. For since a sphere is equal to a cone, whose base is the surface, and its altitude the radius of the sphere ; the surface of the sphere is had, by dividing its solidity by a third part of its semi-diameter :

If therefore the diameter of the circle be 100, the area will be 7850 ; consequently the solidity 1570000, which divided by a third of the semi-diameter 100, the quotient will be the surface of the sphere, or 31400 ; which is manifestly quadruple the area of the circle, and hence the following

RULE.

Find the periphery of the circle, described by the radius of the sphere. This being found, multiply it into the diameter ; and the product is the surface of the sphere. Multiply the surface by the sixth part of the diameter, and the product will be the solidity of the sphere.

EXAMPLE.

Supposing the diameter of the sphere, 56, the periphery will be found 175 ; the product of these two multiplied by a sixth part of the diameter, gives the solidity 92239. Or it may be done in the following manner.

Find the cube of the diameter 175616 : then say, as 300 is to 157, so is 175616 to a fourth proportional, viz. 92239, the solidity of the sphere.

Or thus ;

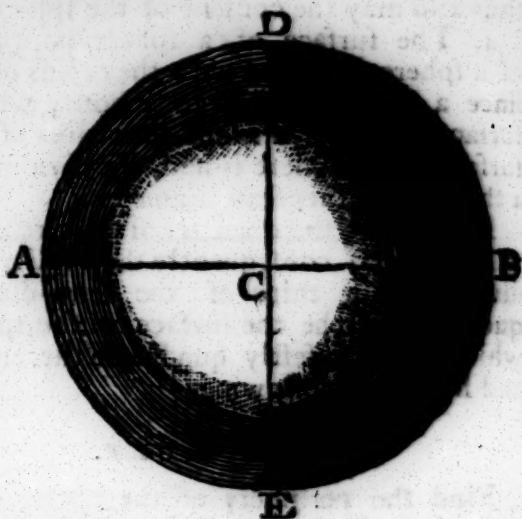
As 21 is to 11, so is the cube of the axis to the solid content. Or, as 1 is to .5236, so is the cube of the axis to the solid content.

K 5

EXAMPLE.

EXAMPLE.

Let ABCDE
beaglobe, whose
axis $AB=DE$
is 20 inches :
the circumse-
rence will be
found to be 62
.832: then by
the first rule,
multiply the cir-
cumference by
the axis, and
the product will
be 1256.64,
which is the
superficial con-
tent in inches :



take a sixth part of that sum, which is 209.44, multi-
ply that sixth part by 20, the axis, and the product will
be 4188.8, the solidity in inches.

Or if you multiply the superficial content by the axis,
and take a sixth part of the product, the answer will be
the same.

Or by this rule: The cube of the axis is 8000, which
multiplied by 11, the product will be 88000, and divided
by 21, the quotient will be 4190.47 the solidity nearly
the same as before.

Or if the cube of the axis be multiplied by .5236,
the product will be 4188.8, the solidity the same as by
the first method, and nearly the same as above. If you
divide 4188.8 by 1728, the quotient will be 2.424 feet.

$$\begin{array}{r}
 62.832 \\
 \underline{20} \\
 6 \overline{) 1256.640} \text{ The superficial content.} \\
 \underline{209.44} \text{ A sixth part.} \\
 \underline{20} \\
 4188.80 \text{ Solidity in inches.} \\
 21 : 11 : 8000 \\
 \underline{11} \\
 21 \overline{) 88000} \overline{) 4190.47} \text{ The content.} \\
 \underline{40} \\
 190 \\
 \underline{100} \\
 160 \\
 \underline{13} \\
 1 : .5236 :: 8000 \\
 \underline{8000} \\
 1728 \overline{) 4188.8000} \overline{) 2.424} \text{ Feet, the solidity.}
 \end{array}$$

It should be observed, that if the axis of a globe be 1. the solidity will be .5236; and if the circumference be 1. the solidity will be .016887.

Of the SPHEROID.

Discip. What do you call a Spheroid, and wherein does it differ from other solids?

Philo. A Spheroid is a solid approaching to the figure of a sphere, not exactly round, but oblong, as having one of its diameters longer than the other, and generated by the revolution of a semi-ellipsis about its axis.

When it is generated by the revolution of a semi-ellipsis about its greater or transverse axis, 'tis then called an oblong or prolate spheroid; and when generated by the revolution of an ellipsis about its lesser or conjugate axis, it is called an oblate spheroid.

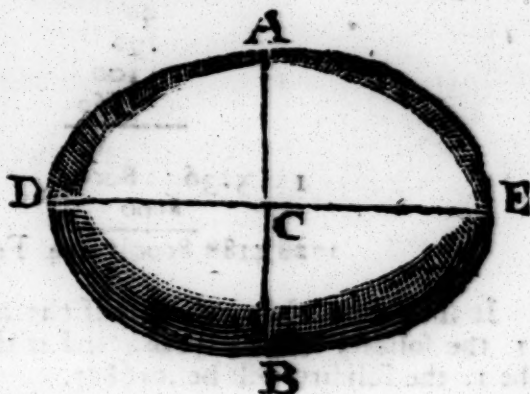
The solid dimensions of a spheroid is $\frac{2}{3}$ of its circumscribing cylinder; or it is equal to a cone, whose altitude is equal to the greater axis, and the diameter of the base, to four times the lesser axis of the generating ellipsis.

Or a spheroid is to a sphere described on its greater axis, as the square of the lesser axis is to the square of the greater. Or it is to a sphere on the lesser axis, as the greater axis is to the less. Hence the solid content of the spheroid may be found by the following RULE.

Multiply the square of the lesser diameter by the length, and that product multiply again by .5236; this last product will be the solidity of the spheroid.

EXAMPLE.

Let AB, the conjugate diameter of the spheroid, ABC DE, be 33 inches, and DE the length or transverse diameter be 55 inches, and let the solidity be required.



OPERATION.

33		59895
<u>33</u>		<u>5236</u>
99		359370
<u>99</u>		179685
1089	Square of the	119790
55	conjugate diameter	<u>299475</u>
5445		31361.0220 Solidity.
<u>5445</u>		
59895		

DEMONSTRATION.

Every spheroid is equal to $\frac{2}{3}$ of a cylinder, whose base is equal to the circle described by the conjugate diameter

diameter of the spheroid, and its height equal to the length of the spheroid.

Of MEASURING FRUSTUMS.

Discip. What do you mean by the word Frustum?

Philo. By the frustum of any figure is meant a piece cut off, and separated from the body. Thus the frustum of a pyramid, or cone, is a part or piece of it cut off usually by a plane parallel to the base.

I shall first shew you the method of measuring the frustum of a pyramid.

The frustum of a pyramid is the remaining part, when the top is cut off by a plane parallel to the base.

To find the solid content of this body, there are several rules; but the two following will be abundantly sufficient for the purpose.

R U L E I.

To the rectangle or product, of the sides of the two bases, add the sum of their squares; that sum being multiplied into one third of the height of the frustum, will give its solidity, if the base be square.

Or thus, which is the same in effect.

Multiply the areas of the two bases together, and extract the square root out of the product; that multiplied by one third of the height, will give the solidity of any frustum, either square or multangled.

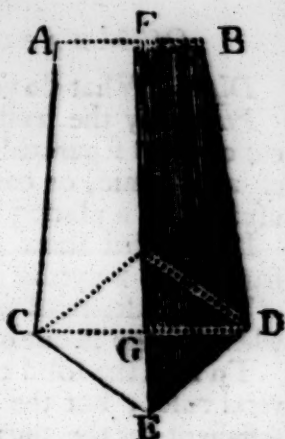
R U L E II.

Add one third part of the square of their difference to the rectangle of the sides of two bases; and that sum being multiplied into the height, will, if the bases be square, produce the solidity: But if they be triangular, or multangular, the said rectangle of the sides, with the third part of the square of their difference, will be the square of a mean side; and the square root thereof will be such a mean side, as will reduce the tapering solid to a prism equal to it.

As for Example.

Let ABCDEF be the frustum of a square pyramid, the side of the greater base 18 inches, and the side of the lesser base 12 inches, and the height FG 18 feet; what is the solidity of it?

First multiply the two sides together, 18 by 12, and the product will be 216; and the difference of the sides is 6, the square of which is 36; a third part of which is 12; which being added to 216, the sum is 228 inches, the area of a mean base; which being multiplied by 18 feet, the length, the product will be 4104. This being divided by 144, the quotient will be 28.5 feet, the content required.



Or by the first Rule thus :

The square of 18 is 324, and the square of 12 is 144, and the rectangle of 18 by 12 is 216 : The sum of these three is 684, which multiplied by 6, the product will be 4104; which divided by 144, the quotient will be 28.5 feet, the same as before.

The Operation both Ways.

$$\begin{array}{r}
 18 \quad 6 \text{ Diff.} \\
 12 \quad 6 \\
 \hline
 216 \quad 3)36 \text{ (Squ.} \\
 12 \text{ add.} \\
 \hline
 228 \text{ the sum.} \\
 18 \text{ the height} \\
 \hline
 1824 \\
 228 \\
 \hline
 144)4104(28.5 \\
 288 \\
 \hline
 1224 \\
 1152 \\
 \hline
 \cdot\cdot 720 \\
 \cdot\cdot 720 \\
 \hline
 \cdot\cdot\cdot
 \end{array}$$

$$\begin{array}{r}
 18 \quad 12 \\
 18 \quad 12 \\
 \hline
 \left\{ \begin{array}{l} 324 \text{ Sq. } 144 \text{ Square} \\ 144 \\ 216 \end{array} \right. \\
 \hline
 684 \text{ the sum} \\
 6 \text{ 3d of the height} \\
 144)4104(28.5 \text{ feet.} \\
 288 \\
 \hline
 1224 \\
 1152 \\
 \hline
 \cdot\cdot 720 \\
 720 \\
 \hline
 \cdot\cdot\cdot
 \end{array}$$

By feet and inches thus :

	F. I. S.	
Multiply	1 : 6 : 6	Or thus ;
by	1 6	F. I.
Product	1 : 6	23 Sq. of the greater.
Add	0 : 1	16 the Rectangle.
Multiply	1 : 7	10 Square of the less.
by	18 : 0 height	49 Trip. of a mean area
	18 : 0	60 a third of the height.
	9 : 0	
	1 : 6	28 6
Content	28 : 6	

To find the superficial Content.

The perimeter of the greater base is 72, and the perimeter of the lesser base is 48; add both the perimeters together, the sum will be 120; the half of this is 60; which being multiplied by 18 feet, the product will be 1080; this being divided by 12, the quotient is 90 feet,

feet, to which add the two bases 2.25 feet, and one foot, the sum will be 93.25 feet, the whole superficial content. See the following

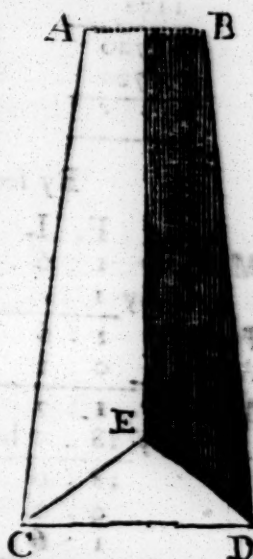
OPERATION.

$$\begin{array}{r}
 18 \ 12 \\
 \underline{4 \ 4} \\
 72 \ 48 \\
 \underline{48} \\
 2)120 \\
 \underline{60}
 \end{array}$$

$$\begin{array}{r}
 18 \text{ Height} \\
 \underline{60} \\
 12)1080(\\
 \underline{90 \text{ Feet}} \\
 2.25 \text{ the greater base} \\
 1 \text{ the lesser base} \\
 \underline{93.25 \text{ Sum.}}
 \end{array}$$

Again, let ABCDE be the frustum of a triangular pyramid, each side of the greater base 30 inches, and each side of the lesser base 10 inches, and the length 20 feet: what is the solid content?

By the second rule multiply 30 by 10, and the product will be 300; and the difference between 30 and 10 is 20; which being squared, make 400, a third part of which is 133.333; which being added to 300, the sum is 433.333; and this being multiplied by .433, the product will be 186.633189, &c. the area of a mean base; and that multiplied by 20 feet, the length, the product will be 3732.66378. This divided by 144, the quotient will be 25.92127 feet, the solidity.



To find the superficial Content.

The perimeter of the greater base is 90, and the perimeter of the lesser base is 30; the sum of both is 120, and the half is 60, which being multiplied by 20 feet, the product will be 1200; which being divided by 12, the quotient will be 100; to which add the sum of the

the two bases 3.33 feet, and the sum will be 103.33 feet, the whole superficial content.

Of the FRUSTUM of a CONE.

A frustum of a cone is the remainder of that solid, when the top end is cut off by a plane parallel to the base.

The solid content of it therefore is found in the same way as the frustum of a pyramid, regard being had to the nature of the lower and upper base, which here are circles.

R U L E I.

To the rectangle of the diameters of the two bases, add the squares of the said diameters, and multiply the sum by .7854, the product will be the triple of a mean area; which multiplied by one third of the perpendicular height, that product will be the solid content.

Or if you multiply the areas of the greater and lesser bases together, and out of the product extract the square root, and add the square root and two areas together, and multiply the sum by one third of the perpendicular height, the product will be the solid content.

R U L E II.

To the rectangle of the greater and lesser diameters, add $\frac{1}{3}$ part of the square of their difference, and multiply the sum by .7854, the product will be a mean area, which multiplied by the perpendicular height, the product will be the solidity.

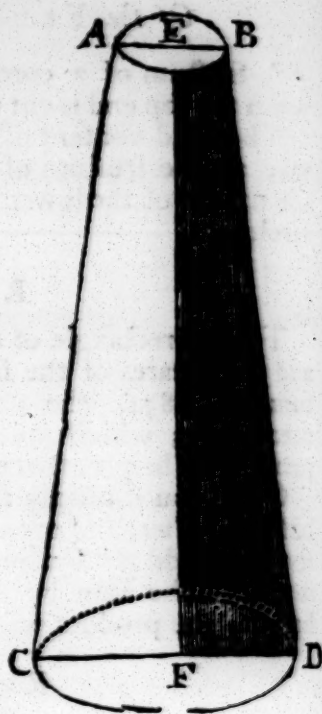
E X A M P L E.

EXAMPLE.

Let ABCD be the frustum of a cone, whose greater diameter CD is 20 inches, and the lesser diameter AB 10 inches, and the height 20 feet, what is the solid content?

OPERATION.

Multiply 20 by 10, and the product will be 200; and the difference between 20 and 10 is 10, whose square is 100, a third part of which is 33.33. which added to 200, the sum will be 233.33. This being multiplied by .7854, the product will be 183.257382, which being divided by 144, the quotient will be 1.273315 feet, the area of the mean base, which multiplied by 20 feet, the height, the product is 25.4663 feet, the solid content.



To find the superficial Content.

You will find the circumference of the greater base to be 62.832, and that of the lesser base 31.416, the sum of both is 94.248, the half sum is 47.124; this multiplied by 20 feet, the product is 944.28; this divided by 12, the quotient is 78.54 feet, the curve-surface; to which add the sum of the areas of two bases 2.726 feet, the sum is 81.266 feet, the whole superficial content.

Of a PRISMOID.

Discip. What do you mean by a Prismoid?

Phil.

Philo. By a prismoid is meant the frustum of a rect-angled pyramid, when both bases are parallel to one another but disproportional.

The content of this solid may be found by the following

R U L E.

Add to the greatest length, half the lesser length, and multiply that sum by the breadth of the greater base, and reserve the product.

Then to the lesser length, add half the greater length, and multiply the sum by the breadth of the lesser base; add this product to the reserved product, and multiply that sum by a third part of the height, the product will be the solid content.

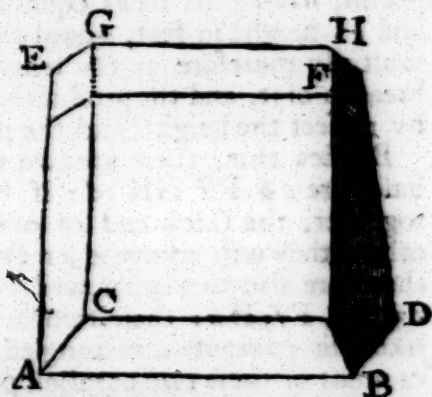
E X A M P L E.

Let ABCDEFGH be a prismoid given, the length of the greater base AB 40 inches, and its breadth AC 20 inches; the length of the lesser base GH 35, its breadth 15 inches, and the height 10 feet: What is the solid content?

To the greater length AB 40, add half the lesser length GH 17.5 the sum will be 57.5, which being multiplied by 20, the greater breadth, the product will be 1150, which reserve.

Then, to GH 35, add half AB 20, and the sum will be 55; which multiply by 15, the lesser breadth, and the product will be 825: To this product add 1150, the reserved product, and the sum will be 1975; which being multiplied by 3.33, third part of the height, the product will be 6576.75; this product divided by 144, the quotient will be 45.67 feet, the solid content required.

To



To prove this RULE.

Let the solid be supposed to be cut into pieces, so as to make it capable of being measured by the foregoing Rule

Let ABCD represent the greater and EF GH the lesser base; and let the solid be supposed to be cut off by the lines ac , bd , ef , gh ; from the top to the bottom; so will there be a parallelo-



pedon, having its bases equal to the lesser base EFGH, and its height 10 feet, equal to the height of the solid: multiply therefore 35 the length of the base by 15, the breadth of it, and the product will be 525; multiply this by 10 feet the height, and the product will be 5250.

Besides this, there are two wedge-like pieces, whose bases are ab EF GH cd ; if these two pieces are laid together, the thick end of one to the thin end of the other, they will compose a rectangled parallelopipedon; there are also two other wedge-like pieces whose bases eg , Ea , Ff , Hh ; these two laid together as before, will likewise compose a rectangled parallelopipedon. The content of these two parallelopipedons must be found in the same manner as that of other parallelopipedons; and their content, added to the above product, will show the solid content of the whole.

To find the superficial Content.

Half the perimeter of the greater base is 60, and half the perimeter of the lesser base is 50; which being added together, the sum is 110; which being multiplied by 10 the height, the product will be 1000: this product divide by 12, and the quotient will be 83.33 feet;

feet; to which add the superficial area of the two bases 9.3 feet, and the sum will be 92.63 feet, the whole superficial content.

There are several other frustums belonging to different solids; but as they are of very little use, except in gauging, I shall not at present enumerate them; but reserve them till I come to apply these rules to the finding the solid contents of various casks, &c. when I shall take care to explain them in the easiest and most conspicuous manner.

Discip. I do not doubt your care and attention to my interest: I have already so greatly profited by your instructions, that my obligations to you are too great, for me ever to be able to repay them: and you are still studying every method in your power to increase their weight.

Philo. Perhaps the pleasure I receive from seeing you improve so greatly in your learning, is at least equal to yours, and it will be the greatest satisfaction to see you, when I have finished my instructions, readily measure any body whether superficies or solid, with accuracy and dispatch.

Of the regular SOLIDS.

Discip. What do you mean by regular solids?

Philo. By regular solids is understood all those bodies that are contained under regular and similar planes, alike situated, and equally distant from the center.

Discip. How many regular solids are there?

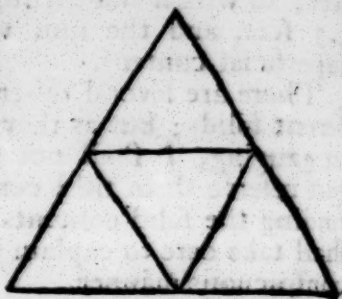
Philo. There are only five: the Tetraëdron; the Hexaëdron; the Octaëdron; the Dodecaëdron; and the Icosaëdron.

Discip. What is a Tetraëdron?

Philo. A Tetraëdron is contained under four equal and equilateral plane triangles; forming four solid angles, each consisting of three triangles; and having six linear edges, and twelve plane angles.

But

But you will more easily form an idea of the tetraëdron by drawing a figure, like that in the margin, on paste-board, or any other pliable substance ; for if you cut the lines half through, turn the parts up, and glue them together, they will form a complete tetraëdron ; and as this is very easily done, I would recommend it to your attention.

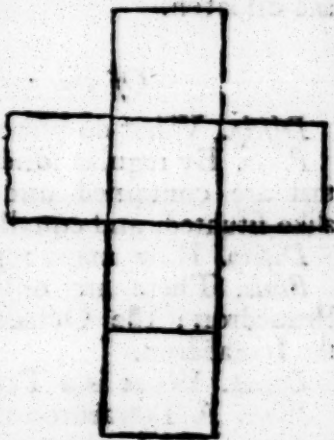


Discip. You may depend upon my care to do every thing that is recommended by you ; especially when the performance has any tendency to facilitate my comprehending your instructions in a more complete manner.

Phil. I do not doubt your care ; nor should I have mentioned this particular, had it not tended to illustrate the subject in the plainest and most conspicuous manner. But I must now return to the explanation of the rest of the regular bodies.

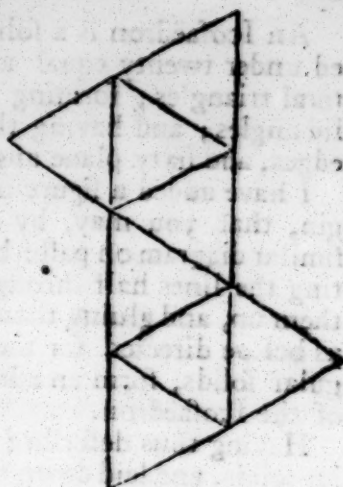
A Hexaëdron is a solid body contained under six equal squares, forming eight solid angles, each consisting of three squares ; and having twelve linear edges and twenty four plane angles.

A similar figure to that in the margin being drawn on paste-board, cut half thro' in the lines, folded up, and glued together, will give you an adequate idea of an hexaëdron.



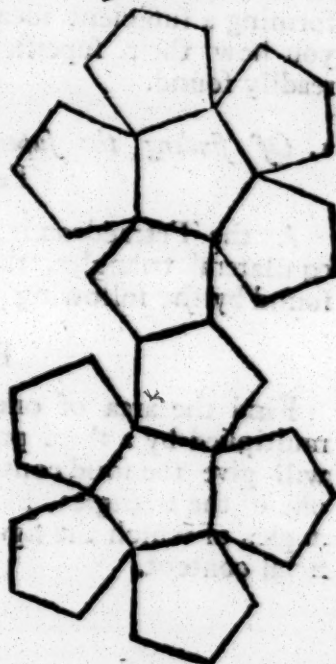
An Octaëdron is a solid contained under eight equal and equilateral plane triangles; forming six solid angles, each four triangles; having twelve linear edges, and twenty-four plane angles.

The figure in the margin drawn on paste-board, the lines cut half thro' folded up, and glued together, will form the figure of an octaëdron.



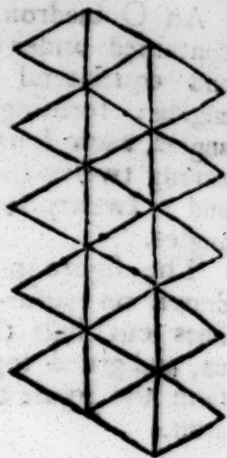
A Dodecaëdron is a solid figure contained under twelve equal, equilateral, and equiangular pentagons; forming twenty solid angles, each consisting of three pentagons, and having thirty linear edges, and sixty plane angles.

If you draw a figure on paste-board, similar to that in the margin, cut the lines half through, fold the pentagons up, and glue the whole together; you will have the true figure of a Dodecaëdron.



An Icosaëdron is a solid contained under twenty equal and equilateral triangles; forming twelve solid angles; and having thirty linear edges, and sixty plane angles.

I have added a figure in the margin, that you may, by drawing a similar diagram on paste-board, cutting the lines half through, folding them up, and gluing them together, as before directed for the other regular solids, form an adequate idea of the Icosaëdron.



Having thus described the regular solids, and laid down methods for forming a sufficient idea of them, it remains to shew you how their superficial and solid contents may be readily found.

1. *Of finding the superficial and solid Content of the Tetraëdron.*

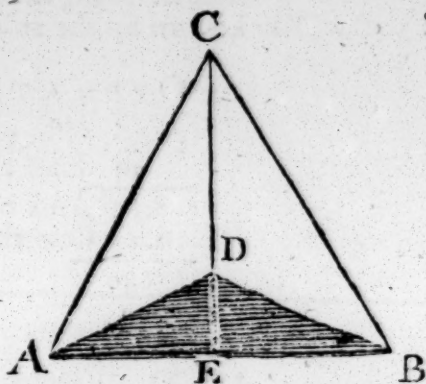
As the Tetraëdron is contained under four equal and equilateral triangles, the solid content will be readily found by the following

R U L E.

Find the area of one of the triangles. This area multiplied by a third part of the perpendicular height, will give the solid content required: And the area of one of the triangles, multiplied by the number of triangles of which the figure consists, will be the superficial content.

Let

Let ABCDE represent a Tetraëdron composed of four triangles, each equal to ABC: let $AB=AC=CB$, be 60 inches, and CD the perpendicular height of the figure be 51.96 inches and let the solid content of the Tetraëdron be required.



I have already pag.

17. shewn the method of finding the area of triangles, when their bases and perpendiculars are given; but as the triangles which form a Tetraëdron, are equilateral, or have their three sides equal to one another, it will be necessary to shew you, my dear Discipulus, how to find the perpendicular of an equilateral triangle, and this is easily done in the following manner. Square one of the sides of the triangle, as AB; and also the half of it, as AE. Subtract the lesser from the greater and extract the square root out of the remainder; and that root will be the perpendicular required.

$$\text{Thus } 60 \times 60 = 3600$$

$$\text{And } 30 \times 30 = 900$$

$$\hline 2700 \text{ (51.96 The perp.)}$$

$$25$$

$$\hline 101).200$$

$$101$$

$$\hline 1029).9900$$

$$9261$$

$$\hline 10386) \cdot 63900$$

$$62316$$

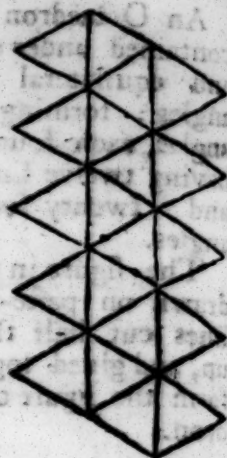
$$\hline \cdot \cdot 584$$

L

The

An Icofaëdron is a solid contained under twenty equal and equilateral triangles; forming twelve solid angles; and having thirty linear edges, and sixty plane angles.

I have added a figure in the margin, that you may, by drawing a similar diagram on paste-board, cutting the lines half through, folding them up, and gluing them together, as before directed for the other regular solids, form an adequate idea of the Icofaëdron.



Having thus described the regular solids, and laid down methods for forming a sufficient idea of them, it remains to shew you how their superficial and solid contents may be readily found.

1. *Of finding the superficial and solid Content of the Tetraëdron.*

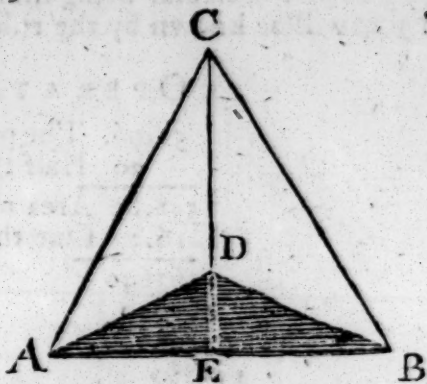
As the Tetraëdron is contained under four equal and equilateral triangles, the solid content will be readily found by the following

R U L E.

Find the area of one of the triangles. This area multiplied by a third part of the perpendicular height, will give the solid content required: And the area of one of the triangles, multiplied by the number of triangles of which the figure consists, will be the superficial content.

Let

Let ABCDE represent a Tetraëdron composed of four triangles, each equal to ABC: let $AB=AC=CB$, be 60 inches, and CD the perpendicular height of the figure be 51.96 inches and let the solid content of the Tetraëdron be required.



I have already pag.

17. shewn the method of finding the area of triangles, when their bases and perpendiculars are given; but as the triangles which form a Tetraëdron, are equilateral, or have their three sides equal to one another, it will be necessary to shew you, my dear Discipulus, how to find the perpendicular of an equilateral triangle, and this is easily done in the following manner. Square one of the sides of the triangle, as AB; and also the half of it, as AE. Subtract the lesser from the greater and extract the square root out of the remainder; and that root will be the perpendicular required.

$$\text{Thus } 60 \times 60 = 3600$$

$$\text{And } 30 \times 30 = 900$$

$$\hline 2700 \text{ (51.96 The perp.)}$$

$$25$$

$$\hline 101).200$$

$$101$$

$$\hline 1029).9900$$

$$9261$$

$$\hline 10386).63900$$

$$62316$$

$$\hline \text{..584}$$

L

The

The perpendicular being thus found, the area of the figure will be known by the rule given above.

OPERATION.

$$\begin{array}{r}
 51.96. \text{ The perpendicular} \\
 30 \text{ Half the base.} \\
 \hline
 1558.80 \text{ Area of one triangle} \\
 16.33 \text{ One third of the height.} \\
 \hline
 467640 \\
 467640 \\
 935280 \\
 155880 \\
 \hline
 25455.2040 \text{ The solid content.} \\
 \\
 1558.8 \text{ The area of one triangle} \\
 4 \\
 \hline
 6235.2 \text{ The superficial content.}
 \end{array}$$

2. *Of finding the solid and superficial content of the Hexaëdron.*

The Hexaëdron is the same with what is otherwise called a cube, which I have already taught you how to measure in page 129, and therefore need not repeat it here.

3. *Of finding the solid and superficial content of the Octaëdron.*

The Octaëdron being contained under eight equal and equilateral triangles, or two quadrangular pyramids joined together by their bases, the solid content will be found by the following

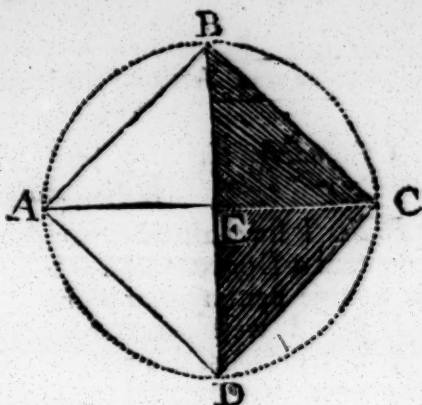
RULE.

Multiply the area of the base by a third part of the length of both pyramids, and the product will be the solid content required.

EXAMPLE

EXAMPLE.

Suppose ABCDE
be an Octaëdron,
whose side $AB=BC$
 $CD=DA$ is 60 in-
ches; what is the so-
lid and superficial
content?



OPERATION.

14.1421	One third of the height.
3600	Area of the square base.
84852600	
424263	
50911.5600	The solid content required.

The superficial content is easily found by the method laid down in the first problem.

OPERATION.

51.96	Perpendicular of one triangle.
30	Half the base.
1558.80	
8	Numb. of triangles
12470.40	Superficial area.

Or because the Octaëdron is composed of eight equilateral triangles, and the Tetraëdron of four; therefore the superficial area of the former is double to that of the latter.

OPERATION.

$$\begin{array}{r}
 6235.2 \text{ The superficial area of the Tetraëdron} \\
 \underline{2} \\
 12470.4 \text{ Area of the Octaëd. the same as before.}
 \end{array}$$

4. Of finding the solid and superficial content of the Dodecaëdron.

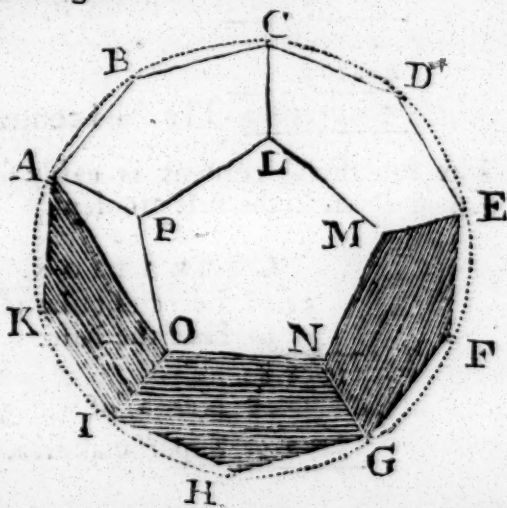
Philo. I have already, my dear Discipulus, informed you, that a Dodecaëdron is a solid body contained under twelve pentangular planes.

Discip. I well remember it; and have, pursuant to your instructions, drawn the figure on paste-board, you delineated for me, folded up the pieces, and by that means, procured an exact Dodecaëdron in miniature.

Philo. You have acted extremely right; and you will now have no difficulty in finding the superficial and solid content of this figure.

In order to this, let ABCDEFGHIKLMNOP represent a Dodecaëdron each side of which is equal to 60 inches; and let both the solid and superficial content be required.

The former or solid content of this figure is composed of twelve pentangled pyramids, whose vertices all meet in the center of the figure, and consequently, the first demand in the question may be easily answered by the following



RULE.

R U L E.

Find the solidity of one of those pyramids ; multiply that result by 12, and the product will be the solidity of the Dedecaëdron required.

The altitude of one of the pentangular pyramids, will be found to be 66.81095 ; and the perpendicular of the pentagon 41.29146.

$$\begin{array}{r}
 41.29146 \text{ Perpendicular} \\
 360 \text{ Half the sum of the sides.} \\
 \hline
 24778760 \\
 12387438 \\
 \hline
 14864.92560 \text{ Area of the pentagon.} \\
 11.27032 \text{ A third part of } 66.81095. \\
 \hline
 2972985120 \\
 4459477680 \\
 103254479200 \\
 2972985120 \\
 1486492560 \\
 1486492560 \\
 \hline
 167524.4682881920 \text{ Content of one pyramid.} \\
 12 \text{ Number of pyramids.} \\
 \hline
 2012936.194583040 \text{ Solidity of the Dedecaëdron.}
 \end{array}$$

The superficial content is easily had by finding the superficial area of one of the pentagons, and multiplying that result by 12, the number of pentagons that compose the superficies of the Dedecaëdron.

O P E R A T I O N.

$$\begin{array}{r}
 14864.92560 \text{ Area of the pentagon.} \\
 12 \\
 \hline
 178379.10720 \text{ Superficial area required.}
 \end{array}$$

5. *Of finding the solid and superficial content of the Icosaëdron.*

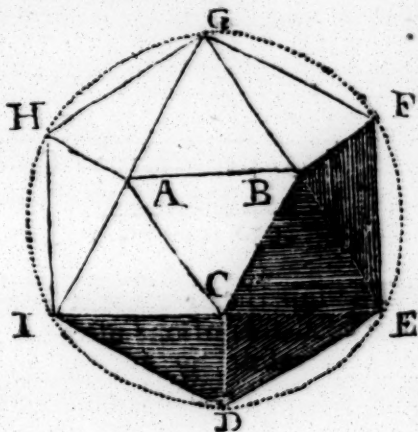
The Icosaëdron, as I have already observed, is a solid body contained under twenty equal and equilateral triangles; and consequently, its solidity is composed of twenty equilateral triangular pyramids, with their vertices all joining in the center. The solid content therefore is readily found by the following

R U L E.

Find the content of one of these triangular pyramids; multiply that result by 20, and the product will be the solid content required.

E X A M P L E.

Let ABCDEFGHI be an Icosaëdron, each side of which is 60 inches; and let both the solid and superficial content be required.



O P E R A T I O N.

51.9612	The perpendicular of the triangle
<u>30</u>	Half the side
1558.8360	Area of one triangle.
<u>20</u>	
31176.7200	Superficial content.

Then

Then

15.115228 One third alt. of the pyramid.

1558.286 Area of one triangle.

90691368

45345684

120921824

120921824

75576140

75576140

15115228

23562.161554608

20

47124.3231092160 The solidity.

As there is seldom any occasion for finding the solid content to so great a degree of accuracy, the major part of the decimals may be omitted; and the result will give the content sufficiently near the truth in actual mensuration. Or you may revert the multiplier as taught in page 25 of the introduction; and by that means obtain an answer to the question with much fewer figures.

But as these calculations are often very troublesome, and very subject to errors, I have added the following proportions of the five regular bodies inscribed in the same circle, taken from Peter Herigon. *Cursus Math.* vol. 1. p. 779, and Barrow's *Euclid*, lib. xiii.

The diameter of the sphere being 2.

The circumference of the greatest circle will be

		6.28318
Superficies of the greatest circle	_____	3.14159
Superficies of the sphere	_____	12.56637
Solidity of the sphere	_____	4.1879
Side of the Tetraëdron	_____	1.62299
Superficies of a Tetraëdron	_____	4.61188
Solidity of a Tetraëdron	_____	0.15132
Side of a cube or Hexaëdron	_____	1.1547
Superficies of the Hexaëdron	_____	8.
L 4.		Solidity

Solidity of the Hexaëdron	————	1.5361
Side of an Octaëdron	————	1.41429
Superficies of the Octaëdron	————	6.9282
Solidity of the Octaëdron	————	1.33333
Side of the Dodecaëdron	————	0.71364
Superficies of the Dodecaëdron	————	10.51462
Solidity of the Dodecaëdron	————	2.78516
Side of the Icosaëdron	————	1.05146
Superficies of the Icosaëdron	————	9.57454
Solidity of the Icosaëdron	————	2.53615

If one of these five regular bodies were required to be cut out of a sphere of any other diameter, it will be as the diameter of the sphere 2 is to the side of any one solid inscribed in the same (suppose the cube 1.1547) so is the diameter of any other sphere (suppose 8) to 9.2376, the side of the cube inscribed in this latter sphere.

Let dr be the diameter of any sphere, and $da = ab = br$ one third of that diameter. Erect the perpendiculars ae , cf , and bi , and draw de , df , er , fr , and ir . Then will

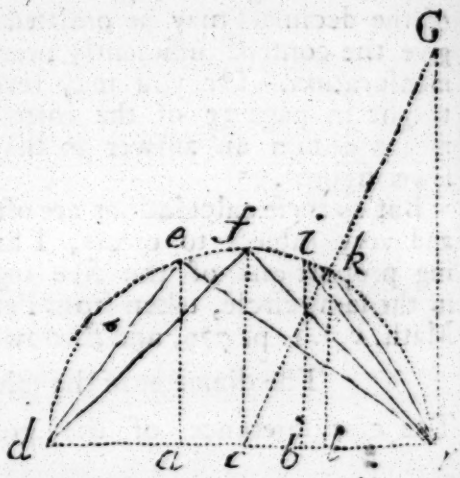
1. re , be the side of the Tetraëdron.

2. $df = rf$ the side of the Hexaëdron.

3. de the side of the Octaëdron.

4. Cut de in extreme and mean proportion in b , and db will be the side of the Dodecaëdron.

5. Let the diameter dr be set up perpendicularly at r , and from the center d , to its top G , draw the line dG , cutting the circle in k , let fall the perpendicular kl ; then is lr the side of the Icosaëdron.



B O O K IV.

*The Application of the foregoing Rules to the Practical
Menfuration of Solid Bodies.*

Philo. **H**AVING, my dear Discipulus, explained the theoretical part of the menfuration of folids; and traced out both by precepts and examples the path that muft be purfued in all inquiries of this kind; it remains that I now fhew you the beft methods of applying thefe rules to practice.

Discip. The trouble you have already taken has laid me under the higheft obligations: but I well know, that the only return I can poffibly make, is to listen to your inftructions, and, by that means, become a mafter of thofe branches of ufeul learning you have kindly undertaken to explain.

Philo. You have rightly gueffed, Discipulus: all I require is the clofeft attention; for by that means, you will afford me the greateft fatisfaction, becaufe you will reap the intended benefit of my inftructions.

S E C T. I.

Of PRACTICAL GAUGING.

Discip. What am I to underftand by practical gauging?

Philo. By practical gauging, is meant the art of finding the quantity of liquor contained in any cask, veffel, &c. And as thefe are of various forms, and confequently require various methods of computation, I fhall begin with the eafieft.

To find the Area of any right-lined Superficies in Gallons.

PROBLEM I.

To find the area of any square tun, back or cooler, &c. either in ale, wine, or corn gallons.

RULE.

Multiply the given length, or breadth, they being in this case equal to each other, into itself, and the product will be the area in inches; then divide that area by 282, 231, or 268.8, and the quotient will be the area required.

EXAMPLE.

Suppose the side of a square tun, back or cooler, be 124.5 inches; what will its area be in gallons?

First 124.5 multiplied by 124.5 gives 15500,25, the area in inches.

Then 282) 15500,25 (54,96, &c. the area in ale gallons.

And 231) 15500,25 (67,10, &c. the area in wine gallons.

Or 268.8) 15500,25 (57,66, &c. the area in corn gallons.

As the operations by division are generally considered as more difficult than those of multiplication, I will shew you, my dear, Discipulus, how you may readily change any divisor into a multiplier; and this requires nothing more than to divide unity or 1. by the divisor you are desirous of changing into a multiplier, which will produce the same answer.

Thus 282) 1,000000(0,003546 the multiplier for ale gallons.

231) 1,000000(0,004329 the multiplier for wine gallons.

268.8) 1,000000(0,0037202 the multiplier for corn gallons.

Hence

Hence you will easily find that 15500,25 multiplied by 0.03546, will give 54.96, the area in ale gallons as before, and so on for the rest.

PROBLEM II.

To find the area of any tun, back or cooler, whose form is that of a right-angled parallelogram, in ale gallons, &c.

RULE.

Multiply the length by the breadth, and the product will be the area in inches. This area being divided by the proper divisors, or multiplied by the equivalent multipliers, will give the solid content in ale, wine, or corn gallons.

EXAMPLE.

Suppose the length of a brewer's tun, back, or cooler be 217,5 inches, and its breadth 85,6 inches, what will its area be in ale or beer gallons?

First 217.5 multiplied by 85.6 gives 18648 the area in inches.

Then $282 \overline{)18648}$ 66,12, &c.

Or, $18648 \times 0,003546 = 66,12$, &c. the area required, &c.

PROBLEM III.

To find the area of any triangular tun, back, or cooler, in ale gallons, &c.

RULE.

Multiply the base by half the perpendicular, or half the base by the perpendicular, and the product will be the area required, and may easily be reduced to ale, wine, or corn gallons, in the same manner as before.

EXAMPLE.

If the length of the base of a triangular cooler be 86,4 inches; and its perpendicular breadth be 57 inches; what will be its area in ale gallons?

First, 86,4 the length multiplied by 28,5 half the perpendicular, gives 2462,4, the area in inches.

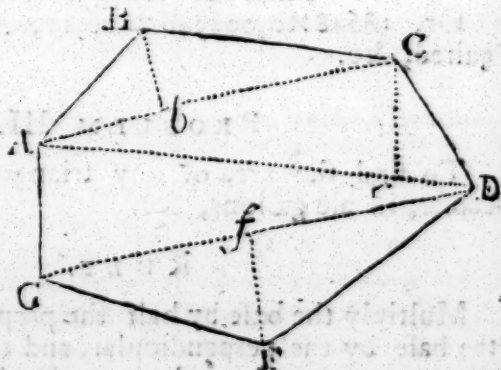
Then $282(2462,4(8,73, \&c.$

Or $24624 \times 0,003546 = 8,73, \&c.$ the area in ale gallons.

By proceeding in this manner, you will easily find the area of any tun, back or cooler, whether it be in the form of Rhombus, Rhomboides, Trapezium, or of any other polygon, either regular or irregular, in ale, or beer gallons, &c. If you first divide it into triangles, and then find the areas of those triangles: the sum of those areas being divided, or multiplied by its proper divisor, or multipliers, which will give the area required.

The practical method of dividing any polygonal tun, back, &c. into triangles, is by the help of a chalked line, like that used by carpenters, and may easily be performed in the following manner.

Suppose a brewer's tun, back or cooler to be in the form of the annexed figure ABCDEFG. Let one end of the chalked line be fastened with a nail, in any corner or angle of the back,



as at A; then straining it to the angle at C, strike the diagonal line AC, upon the bottom of the back; and straining it again to the angle D, strike another diagonal line, as AD: and

and so on for the diagonal line GD: Then having marked out all the diagonal, the perpendiculars may be thus found; fasten as before, one end of the chalked line in the angle B, and then by moving it to and fro upon the stretch, find out the nearest distance between the angle at B, and the diagonal line AC, there strike a line, and it will mark out the perpendicular Bb from B to the line AC; proceed in the same manner to mark out the other perpendiculars, which being all drawn upon the bottom of the back, measure them, and each diagonal by a line of inches, &c. This done, the area of the back may be easily computed as directed by the method laid down in the last problem.

Having found the true area of the back or cooler, the next thing will be to discover the true dipping or gauging place in that back, that the true quantity of liquor may be computed or cast up at any depth. This may be done in the following manner.

1. When the bottom of the back is covered all over, of any depth with liquor, then dip in your rule in eight or ten several places, more or less according to the largeness of the back, as remote and equally distant one from another as you well can, noting down the wet inches, and decimal parts of every dip.

2. Divide the sum of all those dips or wet inches by the number of places you dipped in, and the quotient will be the mean of all those dips.

3. Find out such a place by the side of the back, that just wets your rule to the same height with that mean dip, and make a mark there for the true dipping place of that back. Then if any quantity of wort which covers the whole back be dipped or gauged at that place, and the wet inches so taken be multiplied into the area of the back in gallons, the product will shew how many gallons of wort are in the back at that time, provided the sides of the back stand at right angles with its bottom.

SECTION. II.

To find the area of any circular and elliptical superficies in gallons.

1. The periphery of the circle whose diameter is unity or 1. is 3,1416, and its area is 0,7854, see page 27 and 28.

2. The peripheries of all circles are in proportion one to another, as their diameters; and their areas are in proportion to the squares of their diameters.

As for Example; $1 : 3,1416 ::$ the diameters of any circle: to its periphery.

And $1 : 0,7854 ::$ the square of the diameter: to the area.

Upon these two proportions depends the solution of all the common or practical questions about a circle.

PROBLEM.

The diameter of any circle being given, in inches, to find its area in gallons.

RULE.

Multiply the square of the proposed diameter into 0,7854, and the product will be the area in inches. That area being divided by 282, or 231, &c. the quotient will be the area required.

EXAMPLE.

Suppose the given diameter be 54.5 inches, what will be its area in gallons?

First, $54,5 \times 54,5 = 2970,25$ and $2970,25 \times 0,7854 = 2332,83$ the area in inches.

Then $282)2332,83(8,2724$ the area in ale or beer gallons.

And $231)2332,83(10,0988$ the area in wine gallons.

Or $268)2332,83(8,6788$ the area in corn gallons.

But

But these areas in gallons may be much easier found, without calculating the circle's area in inches; by having the square of the diameter of that circle whose area is one gallon; which may be easily found in the following manner.

$0,785398 : 1 :: 282 : 359,05$ the square of the diameter of the circle whose area is 282 cubick inches, that is, one gallon :

And from this proportion will arise the following divisors.

Viz. $0,785398)282,000000(359,05$ will be a divisor for ale gallons.

And $0,785398)231,000000(294,12$ will be a divisor for wine gallons.

Or $0,785398)268,800000(342,24$ will be a divisor for corn gallons.

If the square of the diameter of any circle, be divided by any one of these constant or fixed divisors, the quotient will shew that circle's area in their respective gallons. As for instance, suppose the square of a circle's diameter is 2970,25.

Then $359,05)2970,25(8,2725$ the area in ale gallons.

And $294,12)2970,25(10,0988$ the area in wine gallons.

Or $342,24)2970,25(8,6788$ the area in corn gallons.

But these divisors may be easily turned into multipliers, by dividing the area into inches of that circle whose diameter is 1.

That is, $0,785398$ by 282 ; Or by 231 , &c.

Thus $282)0,785398(0,02785$ the multiplier for ale gallons.

And $231)0,785398(0,003399$ the multiplier for wine gallons.

Or $268,8)0,785398(0,002922$ the multiplier for corn gallons.

These multipliers are the respective areas of a circle whose diameter is 1, and therefore if the square of the diameter of any circle be multiplied with any of these numbers, the product will be that circle's area in gallons of the same name.

Viz. $2970,25 \times 0,002785 = 8,2725$ the area in ale gallons.

And $2970,25 \times 0,003399 = 10,0988$ the area in wine gallons, &c.

Thus, my dear Discipulus, you will observe, that if the diameter of any circle be given in inches, there are three several ways of finding its area in gallons, and all equally true; but that performed by the constant divisors is most commonly practised.

PROBLEM V.

The transverse, or longest diameter, and the conjugate, or shortest diameter of any elliptical superficies being given, to find its area in gallons.

RULE.

Multiply the two diameters, viz. the length and breadth together, and divide their product by 359,05 for ale gallons, or 294,12 for wine gallons, &c. the quotient will be the area required.

EXAMPLE.

Suppose the longest diameter to be 73,5 inches, and the shortest diameter to be 51,6 inches; what will the area be in ale gallons?

First, $73,5 \times 51,6 = 3792,6$, Then $359,05 \overline{)3792,6}$ (10,56 the area in ale gallons.

Or $294,12 \overline{)3792,6}$ (12,89 the area in wine gallons, &c.

PROBLEM VI.

The diameter of any circle, and the versed sine, viz. the height of any segment being given; to find the area of that segment in gallons.

In page 38, I have shewn an easy method of finding the area of any segment of a circle; and must now observe, that if the area in inches found by that method, be divided by 282, or 231, &c. the quotient will be its area

in

in gallons ; and because the area of any segment may be readily found in gallons, I have here added a short table, which will serve very well for common practice, not only to find the area of any segment of a circle in gallons ; but also to find the number of gallons that are either drawn out, or remain in any cylindrick vessel along, or with its axis parallel to the horizon, usually called the ullage of a cask.

A TABLE

A

T A B L E

OF THE

Segments of a circle, whose area is unity or 1. The Diameter being divided by parallel chord lines into 100 equal parts.

V.S. Segm. V.S. Segm. V.S. Segm. V.S. Segm.

1	0,0017	26	0,2066	51	0,5127	76	0,8155
2	0,0048	27	0,2178	52	0,5255	77	0,8262
3	0,0087	28	0,2292	53	0,5382	78	0,8369
4	0,0134	29	0,2407	54	0,5509	79	0,8474
5	0,0187	30	0,2522	55	0,5565	80	0,8576
6	0,0245	31	0,2640	56	0,5762	81	0,8677
7	0,0308	32	0,2759	57	0,5888	82	0,8776
8	0,0375	33	0,2878	58	0,6014	83	0,8873
9	0,0446	34	0,2998	59	0,6140	84	0,8968
10	0,0520	35	0,3119	60	0,6265	85	0,9058
11	0,0598	36	0,3241	61	0,6389	86	0,9149
12	0,0680	37	0,3361	62	0,6514	87	0,9236
13	0,0764	38	0,3406	63	0,6636	88	0,9320
14	0,0851	39	0,3611	64	0,6759	89	0,9402
15	0,0941	40	0,3755	65	0,6881	90	0,9480
16	0,1032	41	0,3860	66	0,7002	91	0,9554
17	0,1127	42	0,3989	67	0,7122	92	0,9692
18	0,1224	43	0,4112	68	0,7242	93	0,9692
19	0,1323	44	0,4238	69	0,7360	94	0,9755
20	0,1424	45	0,4265	70	0,7477	95	0,9813
21	0,1526	46	0,4491	71	0,7593	96	0,9866
22	0,1631	47	0,4618	72	0,7708	97	0,9913
23	0,1738	48	0,4745	73	0,7822	98	0,9952
24	0,1845	49	0,4873	74	0,7934	99	0,9983
25	0,1955	50	0,5000	75	0,8045	100	1,0000

The

The use of this table of segments depends upon the following proportion.

As the diameter of any proposed circle is to 100, (the diameter of the tabular circle) : So is the height of any segment of a proposed circle to a versed sine in the table.

Then if the tabular segment, which stands against that versed sine, be multiplied into the circle's area, either in inches or gallons, the product will be the area of the segment required, of the same name. If the circle's area be inches, the segment will be inches : If gallons, the segment will be gallons.

EXAMPLE.

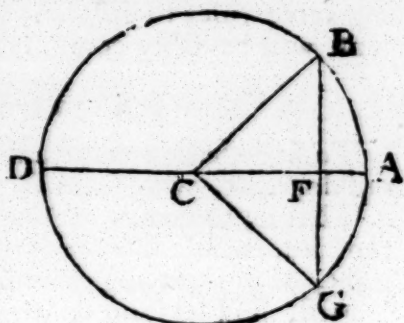
Let the diameter of the given circle be $AD = 62,5$ inches; and the height of the segment sought be $FA = 20$ inches; what will its area be in ale gallons?

First, the area of the whole circle will be 10,8793 ale gallons, and the proportion will stand thus, $62,5 : 100$

$:: 20 : 32$, the versed sine of the table, whose segment is 0,3759.

Then $10,8793 \times 0,3759 = 3,0016$ ale gallons, being the area of the segment $BAGF$, as was required. The same method may be observed for wine gallons, corn gallons, or inches.

Discip. I shall find no difficulty in computing the area of any of these figures; but I should be very glad to know how any superficies can be said to hold any number of gallons: It can contain no more of any fluid than what will cover its surface.



Philo. Your observation is very pertinent and just; but it is always supposed in gauging that the fluid on any of these surfaces is one inch deep.

S E C T. III.

To calculate the contents of such vessels, as are in the form of different solids, as prisms, parallelograms, &c.

Discip. In what manner must I proceed to find the contents of vessels of such various forms?

Philo. Almost every different solid has its own particular rule, which must be followed in computation, as I have already explained in treating of the mensuration of solids, and shall therefore shew the method of applying those rules to different vessels by which their contents will be easily computed.

P R O B L E M VII.

To find the content of any prism, whose sides are parallelograms, what form soever its base is of.

That is, to compute the content, in gallons, of any tun, &c. whose sides are parallelograms standing upright, or at right angles with its bottom.

First, find its solid content in inches by the rule laid down in page 140; then divide that content by 282 or 231, or 268,8, the quotient will shew the content in their respective gallons, viz. in ale, wine, or corn gallons.

Or multiply the content in inches by 0,003546, or 0,004329, &c. those products will be the content in their respective gallons.

Or, you may pursue the following method; first find the true area of the tun's base or bottom; that area being multiplied by the tun's height, will produce the content in gallons, as before.

The work of this problem is so very easy as to require no example.

PROBLEM VIII.

To find the content of any pyramid, in gallons, whose base is bounded by right lines.

Every pyramid is one third part of its circumscribing prism.

Therefore if the area of any pyramid in gallons, be multiplied into one third of its perpendicular height, or if one third of that area be multiplied with the whole height, either of those products will be the content of the pyramid in gallons, &c.

The content of any square pyramid may be easily found in gallons by the following

RULE.

Square the side of its base; and multiply that square by the perpendicular height; then divide that product by $846 = 282 \times 3$ for ale gallons; or by $693 = 231 \times 3$ for wine gallons; or by $806,4 = 268,8 \times 3$ for corn gallons; the quotient will be the content required.

Or if you multiply the above product by 0,001182 for ale gallons, or with 0,001443 for wine gallons, or lastly, with 0,001442, for corn gallons, the result will be the content required.

PROBLEM IX.

To find the content of the frustum of any square pyramid, cut off by a plane parallel to its base.

First, either by rule one or two, given in page 153, find the proposed frustum's solidity in cubic inches. Then divide that content in cubic inches by 282. or 231, &c. and the quotient will be the content of the frustum in the respective gallons.

But from the methods already laid down, there may be easily deduced the following rule, for finding the content of the frustum of any pyramid, what form soever its bases are of, supposing them to be parallel, whether they are alike or unlike.

RULE,

R U L E.

First find the area, then find a geometrical mean between those two areas, that is, multiply the two areas together, and extract the square root out of that product; the sum of those two areas, and their mean being multiplied into one third of the frustum's height, will produce the content required.

E X A M P L E.

Suppose a tun in the form of the lower frustum of a pyramid, whose bases are equilateral triangles. Let the side of the top be 42 inches; the side of the bottom be 63,4 inches; and its depth be 33 inches; what will the content of that tun be in ale gallons?

First, find the area of each base in inches, then find what those areas are in ale gallons. Multiply those two areas together; the square root of their product will be the mean area, &c.

E X A M P L E.

The area of the top is	- 2,71	} Ale gallons.
The area of the bottom is	6,12	
The mean area will be	- 4,07	
	12,90	

Then one third of $12,9 \times 33 = 141,9$ or one third of $12,9 \times 33 = 141,9$ the content required.

P R O B L E M X.

To find the content of any right cylinder in gallons.

That is, to compute the content of any round tun, whose diameters at top and bottom are equal, and at right angles with its sides.

R U L E.

Multiply the square of the diameter into the height: and divide the product by 359,05, or multiply with 0,002785,

0,002785, &c. that quotient, or product, will be the content required.

EXAMPLE.

Suppose the diameter be 42,5 and the height 31,3 inches. First $42,5 \times 42,5 = 1806,25$. And $1806,25 \times 31,5 = 56896,875$. Then $35,05 \mid 56896,875 (15846$ the content in ale gallons, &c.

PROBLEM XI.

To find the content of any cone, or round pyramid in gallons.

Every cone is one third of its circumscribing cylinder, see page 147. Therefore its content may be easily found by the following

RULE.

Multiply the square of the diameter of its base into the perpendicular height; then divide their product by 1077,15 = $359,05 \times 3$ for ale gallons; or by 882,36 = $294,12 \times 3$ for wine gallons, &c. and the quotient will be the content required.

Or if the product be multiplied by 0,000928 = $\frac{2,22785}{3}$ or by 0,0011353 = $\frac{2,2234}{3}$ those products will be the content in their respective gallons.

EXAMPLE.

Suppose the diameter of the base be 42,5 and the perpendicular height be 31,5 inches; what will the content be in ale gallons?

First $42,5 \times 42,5 = 1806,25$. And $1806,25 \times 31,5 = 56896,875$.

Then $1077,15 \mid 56896,875 (52,82$. or $56896,23 \times 0000928 = 52,82$ the content in ale gallons; and in the same manner for wine or corn gallons.

PROBLEM XII.

To find the content of the lower frustum of any cone, in gallons.

That is, to compute the content of any round tun, whose diameter at top and bottom are parallel, but unequal.

The content of such a tun may be found by the rule page 187, whence it will be easy to deduce the following

RULE.

To the triple product of the top and bottom diameters, add the square of their difference; multiply that sum into the height or depth, then divide the last product by 1077,15 for ale gallons, or by 882,36 for wine gallons, the quotient will be the content required.

EXAMPLE.

Suppose the diameter at the top be 52.4 inches; the diameter at the bottom 44,6; and the height 30 inches, what is the content in gallons?

$$\left. \begin{array}{l} \text{First } 52,4 \times 44,6 = 2337,04. \\ \text{And } 2337,04 \times 3 = 7011,12 \\ \text{Also } 52,4 - 44,6 = 7,8 \\ \text{And } 7,8 \times 7,8 = 60,84 \end{array} \right\} \text{Add.}$$

$$\text{The height } 30 \times 7071,96 = 212158,8$$

Then $1077,15(212158,80/196,96 \}$ Content in ale
Or $112158,8 \times 0.000928, = 196,96 \}$ gallons.

You are to proceed in the same manner, either for wine or corn gallons as occasion requires. But if the tun or vessel be not truly circular; that is, if either its top or bottom, or both of them, be elliptical, whether they are alike or unlike is of no consequence, the content of such a tun may be truly found by the general rule, page 187.

PROBLEM XIV.

The axis or diameter of any sphere or globe being given in inches, to find its content in gallons.

Every sphere is two thirds of its circumscribing cylinder, by page 150 and from page 151 it appears, that if the cube of the axis of any sphere, taken in inches, be multiplied into 0,5236, the product will be the content of the sphere in cubic inches. Consequently, if that content be divided by 282, or 231, &c. the quotient will be the content in gallons.

But those two operations of multiplying with 0,5236 and the dividing by 282, or by 231, &c. may be contracted into one.

Thus $282(0,5236(0,001856$ will be a multiplier for ale gallons.

And $231(0,5236(0,002266$ will be a multiplier for wine gallons.

Or $0,5236(282(538,57$ will be a divisor for ale gallons.

And $0,5236(231(441,17$ will be a divisor for wine gallons.

RULE.

If the cube of the axis of any sphere be divided by 538,57, or multiplied with 0,001856, or divided by 441,17, or else multiplied by 0,002266; the quotient or product, will be the content of the sphere in their respective gallons.

EXAMPLE.

Suppose the axis or diameter of a sphere or globe be 22 inches; what is its content in ale gallons?

OPERATION.

Then $22 \times 22 \times 22 = 10648$ and $538,57(10648(19,76$ ale gallons.

Or $10648 \times 0,001856 = 19,76$ ale gallons, the content required.

And so for either wine or corn gallons, as occasion requires.

M

PRO-

PROBLEM XV.

To find the content of any segment of a sphere, in gallons.

If the diameter of the segment's base, and its height are given; the content may be found by the following rules:

RULE I.

To triple the square of half the diameter, add the square of the height; then multiply that sum into the height, and divide the product by 538,57 for ale gallons; or by 441,17 for wine gallons, &c.

But if the axis of the sphere, and the height of the segment are given; the content may be found by the next rule.

RULE II.

From triple the product of the axis into the height subtract twice the square of the height; then multiply the remainder into the height, and divide that product by 538,57, &c.

Either of these rules will produce the content of the segment in gallons.

EXAMPLE.

Suppose the diameter of the segment's base be 28 inches, and its height be 8 inches; what will it contain in ale gallons?

OPERATION.

First $2) 28$ (14. Then (per Rule 1) $14 \times 14 \times 3 = 588$, and $6 \times 6 = 36$. Next $588 + 36 = 624$. Again, $924 \times 6 = 3744$. Lastly, $538,57) 3744$ (695 the content required.

S E C T. IV.

The practical Method of gauging any fixed tun or copper, and making a table to shew what it will hold at every inch deep ; usually called inching of a tun, &c.

The tuns of brewers in general are so fixed, as to lean a little for conveniency of cleansing their drink, which is usually called the drip, or fall of the tun. The drip or fall of any tun, is the hoof of such a solid as that tun is supposed to represent, and may be found by an algebraical process : but the practical method is, to measure into the tun, so much liquor as will just cover its bottom ; by which means you find at once both the fall and a true horizontal line over the bottom of the tun, from which, if the depth of the tun, (viz. the nearest distance from the top of the tun to the surface of the liquor) be set off upon every one of its sides, you will then have a true parallel plane at the top of the tun to that of the liquor.

Then if the sides of the tun are straight from the top to the bottom, take as many dimensions in the two planes, as are necessary to find the true area of each ; and by those two areas, and the above depth, find so much of the tun's content, as is contained between those two planes.

Next to inch that tun ; divide the difference between the top and bottom areas by the depth, and the quotient will be an addend or fixed number, which being added to the lesser area, the sum will be the area of the next inch, and being added to that area, their sum will be the area of the third inch, and so on from inch to inch, until the area of every single inch be found ; the sum of those areas, if the work be true, will be equal to the content found as above. And if the tun's drip or fall be added to the sum of all those areas, that sum will be the whole or full content of the vessel.

From what has been observed it is sufficiently evident, that if 1, 2, 3, or any number of those areas

counted from the bottom be added to the fall, that sum will shew the quantity of liquor that is in the tun, to such a number of wet inches from the bottom, as there are areas added together.

Or if the sum of any number of those areas, being counted from the top, be subtracted from the tun's whole content, the remainder will shew what quantity of liquor is in the tun, when there is such a number of dry inches from the top as there are areas subtracted.

When these directions are well considered there will be no difficulty in forming a table to every wet or dry inch of any tun, whose sides are straight from the top to the bottom; what form soever its bases are of, and whether it stand upon its greater or lesser base.

But if the sides of the tun are not straight from the top to the bottom; then, the best and easiest way will be to divide the tun into several frustums, each of 10 inches deep, and finding the content of every single frustum, by taking the diameters in the middle of every one of those 10 inches; that is, the first diameter at five inches from the top; the second diameter at 15 inches from the top, &c. and multiplying their respective areas by 10, which is done by only removing the separating commas one place forward to the right hand. This being done, if the sum of all those frustums be added to the fall, that sum will be the whole content of the tun.

It must be observed, that if you take the height of the 10 inch frustums in the side of the tun, you must allow for the difference between the slant height, and the perpendicular height in every frustum.

If from the whole content of the tun, you subtract the mean area of the first frustum ten times, and from the remainder subtract the mean area of the second frustum ten times, and from the last remainder subtract the mean area of the third frustum, &c. until there remains nothing but the fall or hoof of the tun; you will have a table that will shew what quantity of drink is in the tun, to any number of dry inches.

This method is likewise used for gauging and inching a brewer's copper. First, Measure into the copper

as much liquor as will just cover its crown : then divide its perpendicular height into frustums, and its sides into four equal parts, that so cross diameters may be taken in the middle of each frustum. But if the copper is much wider at the top than at the bottom, and its sides spheroidical arching ; then instead of taking those mean diameters in the middle of every ten inches, you must take them in the middle of every six inches ; but in every other respect you may proceed as before.

But the quantity of liquor that would cover the crown of the copper may be found without measuring it as above, by supposing the crown to be the segment of a sphere, and the lower part of the copper, wherein the crown rises, to be the frustum of a parabolic conoid : then if the diameter at the top of the crown, and its perpendicular height are given, the quantity of liquor may be found by the following

R U L E.

From the area of the plane at the top of the crown subtract $1\frac{1}{2}$ of the area of the crown's height ; the remainder being multiplied into half the height of the crown, will produce the quantity or number of gallons that will cover the crown.

S E C T. V.

To compute the content of any close cask, viz. butt, pipe, bog-head, &c. in gallons.

In order to perform this difficult part of gauging, the three following dimensions of the proposed cask must be accurately taken in inches, and decimal parts of an inch.

Viz. The bouldge or bung diameter, within the cask.

Either of the head diameters, supposing them both equal.

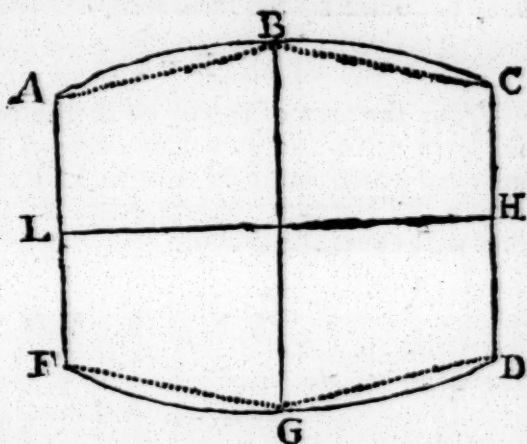
And the length of the cask within.

In taking these dimensions, it must be carefully observed, first, that the bung-hole be in the middle of the cask ; also that the bung-staff, and the staff over against

bung-hole, are both regular or even within. Secondly, That the heads of the cask are equal and truly circular; if so, the distance between the inside of the chine, to the outside of it's opposite staff, will be the head diameter within the cask. Thirdly, With a sliding pair of calipers, take the shortest distance between the outides of the two heads; from that length subtract $1\frac{1}{2}$ inch more, or according to the largeness of the cask, for the thickness of the head; the remainder will be the length of the cask within.

Now, by these dimensions it is natural to suppose, the content of the cask were perfectly limited; but it will be easy to perceive, by the following figure, that the diameters and the length of one cask, may be equal to those of another; and yet one of those casks may contain several gallons more than the other.

For instance, suppose the annexed figure ABCDGF, to represent a cask; then it is plain, that if the outward curved lines A B C, and F G D are the bounds or staves of the cask; it must of course hold



more than if the inner straight or pricked were its bounds or staves; and yet the bung diameter BG, head diameters CD and AF, and the length LH, are the same in both those casks.

From this it plainly appears, that no one certain or general rule can be prescribed, to find the true content of all sorts of casks; and therefore, gaugers usually suppose every cask to be in form of some one of these following solids.

Viz.

- Viz.* { I. The middle zone or frustum of a spheroid.
 II. The middle zone or frustum of a parabolic spindle.
 III. The lower frustum of two equal parabolic conoids.
 IV. The lower frustum of two equal cones.

The method of finding the form of the cask, and computing its content according to that supposed form, is to be performed thus :

I. If the staves of the cask are very curved or arching, as the outward lines of the last figure ; then the cask is supposed to be in the form of the middle zone, or frustum of a spheroid, whose content may be computed by the two following rules.

R U L E I.

To twice the square of the bung diameter, add the square of the head diameter ; multiply that sum into the length, and divide the product by 1077,15. viz. $3,8197 \times 282$ for ale gallons ; and by 882,36. Viz. $3,8197 \times 231$ for wine gallons.

R U L E II.

To twice the area of the bung circle, add the area of the head circle ; multiply their sum into one third of the length, and the product will be the content in their respective gallons.

E X A M P L E I.

Suppose a cask in the form of the middle zone of a spheroid, whose bung diameter is 31,5, head diameter 24,5, and its length 42 inches.

First, $31,5 \times 31,5 \times 2 = 1984,5$ and $24,5 \times 24,5 = 600,25$.

Again, $1984,5 \times 600,25 = 2584,75$. and $2584,75 \times 42 = 108559,5$.

Then $1077,15 \mid 108559,5 \{ 100,78$ the content in ale gallons.

M 4

And

And $882,35 \times 108559,5 (123,07$ the content in wine gallons.

Or thus by the second Rule.

Bung diameter 31,5, twice its circle's area is $5,5270$
 Head diameter 24,5, its circle's area is $1,6718$

The length 42 divided by 3 is 14. Their sum $7,1988$

Then $7,1988 \times 14 = 100,78$ the content in ale gallons.

The content in wine gallons may be found in the same manner.

II. If the staves of the cask are not quite so much curved or arching as was supposed before ; the cask is then taken for the middle frustum of a parabolic spindle ; and its contents may be computed by this

RULE.

To twice the square of the bung diameter, add the square of the head diameter ; from their difference subtract four tenths of the square of the difference of the diameters ; multiply the remainder into the length, and divide the product by 1077,15, &c. as above.

EXAMPLE II.

Suppose the dimensions the same as before. Then $31,5 \times 31,5 \times 2 : + 24,5 \times 24,5 = 2584,75$, and $31,5 - 24,5 = 7$. Again, $7 \times 7 = 49$ four tenths of which $= 19,6$, and $2584,75 - 19,6 \times 42 = 107736,3$. Then $1077,15 \times 107736,3 (100,10$ the content in ale gallons, &c.

III. When the staves of the cask are but very little curved or arching, then it is supposed to be in the form of the frustums of two equal parabolic conoids, abutting or joining together upon one common base at the bouldge, and the content may be found by the following rules.

RULE I.

To the square of the bung diameter, add the square of the head diameter ; multiply their sum into the length, and divide the product by 718,08.

Viz. $2,5464 \times 282$) for ale gallons; or by $588,22$.

Viz. $2,5464 \times 231$) for wine gallons; or thus:

RULE II.

To the area of the bung circle, add the area of the head circle, multiply the sum into half the length, and the product will be the content required.

EXAMPLE III.

With the same dimensions as before.

First, $31,5 \times 31,5 : + 24,5 \times 24,5 = 1591,5$ and $1592,5 \times 42 = 66885$. and $718,08)66885(93,01$ the content in ale gallons.

Or $588,22)66885(113,7$ the content in wine gallons.

IV. If the staves of the cask are straight from the bouldge to the head, as in the inner pricked lines in the last figure; it is then taken for the lower frustums of two equal cones, abutting or joining together upon one common base at the bouldge, and its contents may be computed thus:

RULE.

To the sum of the squares of the head and bung diameters, add their product; then multiply that sum into the length, and divide the last product by $1077,15$. Or by $882,36$, the quotient will be the content, &c.

EXAMPLE IV.

With the same dimensions as before.

First, $31,5 \times 31,5 : + 24,5 \times 24,5 : + 31,5 \times 24,5 = 2364,25$. And $2364,25 \times 42 = 99298,5$. Then $1077,15)99298,5(92,18$, the content in ale gallons, and so on for wine gallons.

Thus you have the methods of computing the true contents of the four solids, in whose forms all casks

are supposed to be ; and by the examples it appears, that of four such casks as have their dimensions all equal, and the same with those above-mentioned, the contents will be as in the margin.

	Ale	Gall.	Differ.
I.	100,78		0,77
II.	100,01		7,00
III.	93,01		0,83
IV.	92,18		

From the disproportion or inequality of these differences, it will be easy to imagine, that there may be several casks whose contents cannot be truly found, according to the supposed forms ; and, therefore, in order to rectify those inequalities, some authors have laid down theorems of their own invention, and others have proposed tables for the same purpose ; but since all that we can attain to is only guess at the truth, the best and easiest way is to be preferred in practice, which is, by finding such a mean diameter as will reduce the proposed cask to a cylinder, thus :

Multiply the difference between the head and bung diameters by 0,7 or by 0,65 or by 0,6, or by 0,55 according as the staves of the cask are more or less arching ; add the product to the head diameter, and the sum will be the mean diameter required.

EXAMPLE V.

With the same dimensions as before. Then the bung diameter less the head diameter is $31,5 - 24,5 = 7$. And

	M. D.	A. G.	Cont.	Dif.
24,5	$7 \times 0,7 = 29,4$ its area	$2,4073 \times 42 = 101,0$		2,39
	$7 \times 0,65 = 29,05$ ———	$2,3504 \times 42 = 98,71$		2,36
	$7 \times 0,6 = 28,7$ ———	$2,2941 \times 42 = 96,35$		2,32
	$7 \times 0,55 = 28,35$ ———	$2,2385 \times 42 = 94,03$		2,32

Hence it may be observed, that the difference between each cask's content is regular, and very near equal ; which plainly shews, that there is not so much room left for error by this method of computing their contents, as by the above forms.

S E C T. VI.

To find what quantity of liquor is either drawn forth, or remaining in any spheroidical cask, usually called the ullage of a cask.

CASE 1. To find what quantity of liquor is in the cask, when its axis is perpendicular to the horizon; viz. when it stands upright upon one of its heads.

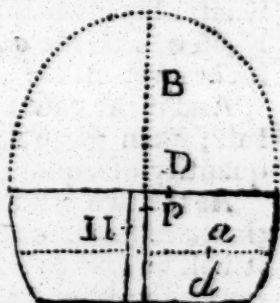
In order to perform this the easiest way, it will be convenient to know how to calculate the area of any circle, betwixt the bung and the head, whose distance from the bung or middle of the cask is given, which may be done in the following manner.

As the square of half the length of the cask : is to the difference between the bung and head areas : : so is the square of any circle's distance from the bung : to the difference between the bung area, and the area of that circle; viz. the area of the liquor's surface.

D E M O N S T R A T I O N.

Let $\left\{ \begin{array}{l} H = \text{half the length of the} \\ \text{cask.} \\ D = \text{half the bung diame-} \\ \text{ter.} \\ d = \text{half the head diame-} \\ \text{ter.} \end{array} \right.$

And $\left\{ \begin{array}{l} P = \text{the distance of any} \\ \text{circle from the bung.} \\ a = \text{half the diameter of} \\ \text{that circle.} \end{array} \right.$



Then according to the common property of the ellipsis, it will be, $BB : DD :: BB - HH : dd$. And $BB : DD :: BB - PP : aa$.

Hence this analogy $HH : DD - dd :: PP : DD - aa$. Then $DD - aa$ being subtracted from DD , will leave aa : but circles areas are in proportion to the squares of their diameters. Therefore, the above analogy is true.

Then, from the bung area, subtract one third part of the aforesaid difference; viz. between the bung area, and the area of the liquor's surface; multiply the remainder by the liquor's distance from the bung, and the product will shew what quantity of liquor is either above, or under half the content of the cask.

EXAMPLE.

Let us suppose a cask of the same dimensions with that in the first example, page 126, and let it be required to find what quantity of liquor, ale measure, is in it, when there is but nine inches wet. Here half the length of the cask is 21 inches, whose square is 441, and the liquor's distance from the bung is $21 - 9 = 12$, its square is 144. The difference between the bung and head areas is 1,0917 ($= 2,7635 - 1,6718$). Then $441 : 1,0917 :: 144 : 0,3564$.

And $2,7635 - 0,3564 = 2,4071$, the area of the liquor's surface.

Again, $3)0,3564(0,1188$. And $2,7635 - 0,1188 = 2,6447$. Then $2,6447 \times 12 = 31,7364$, what the cask wants of being half full. Consequently $50,39 - 31,73 = 18,66$ will be the quantity of liquor in the cask at nine inches wet, in ale gallons.

And if the cask had wanted but nine inches of being full; then $50,39 \times 31,73 = 82,12$ would have been the quantity of liquor in the cask.

As the two first terms, 441 and 1,0917, in the proportion, are fixed, or continue the same for any distance; it will be easy to calculate the areas of all the circles between the bung and head to every inch; and by that means to make a table which will shew what quantity of liquor is either drawn out or remaining in the cask at any depth.

CASE 2. To find what quantity of liquor is in any cask, when its axis is parallel to the horizon; viz. when it lies along.

The truest method of coming to the truth, in all sorts of casks, is by computing the ullage by a table of segments, in the following manner:

1. By

1. By the bung and head diameters find such a mean diameter as you judge will reduce the proposed cask to a cylinder, by the method already laid down; and then find its full content by the preceding examples.

2. From the bung diameter, subtract the mean diameter, and halve their difference, that is, divide it by two.

3. From the wet inches of the proposed ullage, subtract the said half difference, and call it x ; then observe the following proportion:

As the mean diameter : is to 100 (the diameter of the tabular circle) : : so is the last difference, (viz. x) to a versed sine in the table, page 182.

Then if the tabular segment, which stands against that versed sine, be multiplied into the content of the cask; the product will shew the ullage; viz. what quantity of liquor is either in the cask or drawn out.

EXAMPLE.

Take a cask, whose bung diameter is 31,5 inches; mean diameter 29,05; and content 98,71 ale gallons; then suppose there were 10,5 inches wet in it; it is required to find the wet and dry gallons.

Here $31,5 - 29,05 = 2,45$; its half is 1,22. And $10,5 - 1,22 = 9,28$. Then $29,05 : 100 :: 9,28 : 0,319 = V$. line; its segment is 0,2748, And $98,71 \times 0,2748 = 27,12$ the number of wet gallons.

Again, $31,5 - 10,5 = 21$, the dry inches; and $21, - 1,22 = 19,78$. Then $29,05 : 100 :: 19,78 : 0,68$; its segment is 0,7241. And $98,71 \times 0,7241 = 71,48$, the number of dry gallons.

Proof, $71,48 \times 27,12 = 98,6$, is nearly the content of the cask.

C H A P. II.

Of the practical methods of measuring timber, stone, &c.

Philo. THESE branches of mensuration being of the utmost importance, I could wish, my dear Discipulus, that you would be very attentive to the instructions I shall give you, relative to these subjects, because they will be found highly useful on many occasions.

Discip. It is hardly possible for any person, desirous of understanding those branches of learning, to neglect so fair an opportunity of increasing his knowledge, and acquiring a system of instructive lessons, that cannot fail of being highly useful to him during his whole life. For my own part, I value your instructions at so high a rate, that I shall always think the day you were kind enough to take me under your care, as one of the most fortunate in my whole life.

Philo. It gives me great satisfaction to hear you are so fond of your learning: and give me leave to add, that the rules I shall lay down with regard to those practical branches of mensuration, will be so plain and easy to be understood, that you will find no difficulty in putting them in practice, whenever there shall be occasion.

S E C T. I.

Of Measuring Timber.

In order to render this art more intelligible, I shall divide it into three branches. In the first I will shew the method of measuring square or equal sided timber: in the second, that of unequal sided timber: and in the third, I will explain the various methods used by practical measurers for finding the content of round timber, whether it be of the same diameter throughout, or tapering, as trees, &c.

1. *Of measuring square timber.*

Discip. What am I to understand by square timber ?

Philo. By square timber is to be understood, all such pieces as are hewed, or sawed into a true parallel-piped ; or whose breadth and depth are equal to one another. And all these are measured by the following

R U L E.

Find the area of the base, (that is, multiply the breadth by the depth) and multiply that area or product by the length : the result of the last operation will be the content desired.

E X A M P L E I.

Suppose a piece of timber be one foot, nine inches in breadth, the same in depth, and 20 feet long : how many solid feet does it contain, and what will it come to at 1s. 9d. per foot ?

Decimal Solution.

$$\begin{array}{r}
 1.75 \text{ Breadth} \\
 1.75 \text{ Depth} \\
 \hline
 875 \\
 1225 \\
 175 \\
 \hline
 3.0625 \\
 20 \text{ Length} \\
 \hline
 61.2500 \text{ Content.}
 \end{array}$$

As

As one foot : 1s. 9d. = 1.75 :: 61.25 :

$$\begin{array}{r}
 61,25 \\
 \underline{1,75} \\
 30625 \\
 42875 \\
 6125 \\
 \hline
 s. 107,1875 \\
 \underline{12} \\
 d. 2,2500 \\
 \underline{4} \\
 f. 1,00
 \end{array}$$

Ans. The content is 61 feet and a quarter, and the price 5*l.* 7*s.* 2*d.* $\frac{1}{4}$.

By Cross Multiplication.

$$\begin{array}{r}
 1-9 \\
 \underline{1-9} \\
 1-3-9 \\
 \underline{1-9} \\
 3-0-9 \\
 \underline{20} \\
 61-3-0
 \end{array}$$

Discip. I am sorry to interrupt you—but I have often heard workmen mention a *load* of timber ; what do they mean by that term ?

Philo. By a load of timber workmen and measurers mean 40 feet of rough or unhewn timber, and 50 feet of hewn timber ; and this quantity is supposed to weigh a ton, or 20 hundred weight. The reason they give for this difference between a load of hewn or unhewn timber is, that the former is measured very near the truth ; whereas the latter is so far from being so, that what is sold for 40 is about 50 feet ; consequently the weight of each is nearly equal. So that upon the whole, it amounts to the same thing. Hence you may easily reduce the solid content of any piece or pieces of timber to loads by the following

RULES.

RULES.

Divide the feet of hewn timber by 50; the quotient gives the loads.

Divide the feet in rough timber by 40; the quotient gives the loads.

There is, however, some difference with regard to the terms in the King's Yards. They call 40 feet of hewn timber a ton, and 50 feet a load.

EXAMPLE.

Suppose a merchant has seven pieces of square timber to dispose of, at 30s. 6d. per load, of the following dimensions, viz. No. 1. 2 feet, 6 inches the side of the square, and 30 feet long: No. 2. 3 feet, 3 inches the side of the square, and 40 feet long: No. 3. 2 feet 9 inches the side of the square, and 25 feet long: No. 4. 4 feet 2 inches the side of the square, and 18 feet 6 inches long: No. 5. 3 feet 9 inches the side of the square, and 22 feet long: No. 6. 4 feet the side of the square, and 18 feet long: and No. 7. 5 feet 3 inches the side of the square, and 28 feet long. What is the solid content of the whole; and what will it amount to at the above price?

Decimal Solution.

NUMB. I.

$$\begin{array}{r}
 2.5 \\
 2.5 \\
 \hline
 125 \\
 50 \\
 \hline
 6.25 \\
 30 \text{ Length} \\
 \hline
 187.50 \text{ Content.}
 \end{array}$$

NUMB.

NUMB. II.

$$\begin{array}{r}
 3.25 \\
 \underline{3.25} \\
 1625 \\
 650 \\
 \underline{975} \\
 10.5625 \\
 \underline{40 \text{ Length}} \\
 \underline{422.5000 \text{ Content.}}
 \end{array}$$

NUMB. III.

$$\begin{array}{r}
 2.75 \\
 \underline{2.75} \\
 1375 \\
 1925 \\
 \underline{550} \\
 7.5625 \\
 \underline{25 \text{ Length}} \\
 378125 \\
 \underline{151250} \\
 189.0625 \text{ Content.}
 \end{array}$$

NUMB. IV.

$$\begin{array}{r}
 4.166 \\
 \underline{4.166} \\
 24996 \\
 24996 \\
 4166 \\
 \underline{16664} \\
 17355556 \\
 \underline{18,5} \\
 86777780 \\
 138844448 \\
 \underline{17355556} \\
 \underline{321,0777860 \text{ Content.}}
 \end{array}$$

NUMB.

NUMB. V.

$$\begin{array}{r}
 3.75 \\
 \underline{3.75} \\
 1875 \\
 2625 \\
 1125 \\
 \underline{14.0625} \\
 22 \text{ Length} \\
 \underline{281250} \\
 281250 \\
 \underline{309.3750} \text{ Content.}
 \end{array}$$

NUMB. VI.

$$\begin{array}{r}
 4 \\
 \underline{4} \\
 16 \\
 18 \text{ Length.} \\
 \underline{128} \\
 16 \\
 \underline{288} \text{ Content.}
 \end{array}$$

NUMB. VII.

$$\begin{array}{r}
 5.25 \\
 \underline{5.25} \\
 2625 \\
 1050 \\
 2625 \\
 \underline{27.5625} \\
 28 \\
 \underline{2205000} \\
 551250 \\
 \underline{771.7500} \text{ Content.}
 \end{array}$$

Contents.

No. 1.	—187.5
2.	—422.5
3.	—189.0625
4.	—321.077786
5.	—309.375
6.	—288
7.	—771.75
	<u>50)2489.26528(6</u> Content.

As 1 load : 11. 10s. 6d. = 1.5025 : : 497.853105 loads.

$$\begin{array}{r}
 1.5025 \\
 2489265 \\
 995706 \\
 24892650 \\
 497853 \\
 \hline
 748.0241325 \\
 20 \\
 \hline
 .4826500
 \end{array}$$

Answ. The number of loads is 497.853 ; the price 748/.

As I have given the whole operation by decimals at large, I shall leave the solution by duodcimals, or cross multiplication, as an exercise to yourself, fully persuaded that you will find no difficulty in the task : and shall conclude the mensuration of square timber, with proposing the following questions, and giving their answers, without the arithmetical operations.

QUESTION I.

How many solid feet are contained in a piece of timber, when the side of the square is 2 feet 11 inches, and the length 37 feet ?

Answ. 314 feet, 9 inches, 1 twelfth of an inch.

QUEST-

QUESTION II.

Suppose a piece of timber, 1 foot 8 inches the side of the square, and 29 feet 6 inches long; what is its solid content?

Ans. 81 feet, 11 inches, 4 twelfths.

QUESTION III.

If the side of the square of a piece of timber be 4 feet 7 inches, and its length 18 feet; what is its solid content?

Ans. 231 feet 1 inch and a half.

QUESTION IV.

It is required to find the solidity of a piece of timber, the side of the square being 1 foot 7 inches, and the length 31 feet 6 inches.

Ans. 78 f. 11 in. 7. p.

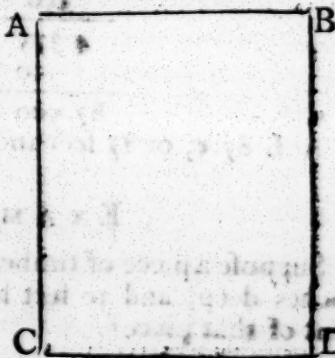
2. Of measuring unequal sided Timber.

Discip. What am I to understand by *unequal sided* timber.

Philo. That the breadth of a piece of timber is greater than its depth, or its depth greater than its breadth; that is, when these two dimensions are unequal.

Thus, if ABCD, represent the base of a piece of timber. and $AC = BD$ be longer than $AB = CD$, and these dimensions are the same through the whole piece, it is said to be unequal sided.

Discip. I am greatly obliged to you for this explanation; and shall now be glad to know how I am to proceed, in order to find the solidity of pieces of timber of this kind.



Philo.

Philo. The whole difficulty of measuring these pieces by tables, consists in reducing the base to a square; that is, in finding the side of a square, whose area is equal to that of a parallelogram, whose dimensions are given: and this difficulty is easily removed by the following

R U L E.

Multiply the two sides together, and extract the square root out of the product: that root will be the side of the square required.

But in order to find the solidity by arithmetical computation, there is not the least necessity for this trouble: you have nothing more to do than to multiply the breadth by the depth, and that product by the length; for the last result will be the solidity required.

E X A M P L E I.

What is the solid content of a piece of timber, whose breadth is 2 feet 6 inches, its depth 1 foot 9 inches, and its length 20 feet?

Decimal Solution.

$$\begin{array}{r}
 1.75 \text{ Depth} \\
 2.5 \text{ Breadth.} \\
 \hline
 875 \\
 350 \\
 \hline
 4.375 \text{ Area of the base} \\
 20 \text{ Length.} \\
 \hline
 87.500 \text{ Solidity.}
 \end{array}$$

Anf. 87.5, or 87 feet and a half.

E X A M P L E II.

Suppose a piece of timber be 4 feet broad, 3 feet 6 inches deep, and 30 feet long, what is the solid content of that piece?

Decimal

Decimal Solution.

$$\begin{array}{r}
 3.5 \text{ Depth} \\
 4 \text{ Breadth} \\
 \hline
 14.0 \\
 30 \text{ Length.} \\
 \hline
 420
 \end{array}$$

Ans. 420 feet.

EXAMPLE III.

If on measuring a piece of timber, I find it 2 feet 3 inches broad, 2 feet 9 inches deep, and 40 feet long; how many solid feet does the piece contain?

Solution by Decimals.

$$\begin{array}{r}
 2.25 \text{ Breadth} \\
 2.75 \text{ Depth.} \\
 \hline
 1125 \\
 1575 \\
 450 \\
 \hline
 6.1875 \\
 40 \\
 \hline
 247.5000
 \end{array}$$

Ans. 247.5 feet.

Dissep. You mentioned, in measuring unequal sided timber by tables, it was necessary to multiply the dimensions of the base together, and extract the square root out of the product, in order to find the side of a square, whose area should be equal to the area of the parallelogram. I should be glad to know what you mean by measuring timber by tables?

Philo. There are several treatises extant, by which you may readily find the solidity of any piece of timber, according to the practical method of measuring without the help of any arithmetical calculations, as I shall explain more fully, when I come to shew you the manner of measuring tapering timber, where I propose
to

to explain the method used in computing those tables, and the use that should be made of them. In the mean I shall pursue the method I have laid down at the beginning of this section.

3. *Of measuring round timber.*

Round timber is of two kinds, first, That where the two ends are equal to each other. Second, That where the two ends are not equal to each other, and therefore called tapering timber.

1. *Of measuring round timber, when both the ends are equal.*

Pieces of round timber, where both the ends are equal, are cylinders, and their solidity may be found by the following

RULE.

Multiply the square of one fourth of the circumference by the length, and the product is the solidity required.

EXAMPLE I.

Let it be required to find the solid content of a piece of round timber, whose circumference is 4.496 feet, and its length 10 feet.

Solution

Solution by Decimals.

1.124 one fourth of the circumference.

$$\begin{array}{r}
 1.124 \\
 \hline
 4496 \\
 2248 \\
 1124 \\
 1124 \\
 \hline
 1.263376 \\
 10 \\
 \hline
 12.633760 \\
 12 \\
 \hline
 7.80512
 \end{array}$$

Anf. 12.63376 feet; or 12 feet, 7 inches, and 8 tenths of an inch.

EXAMPLE II.

What is the content of a piece of round timber, whose circumference is 8 feet 4 inches, and its length 15 feet; supposing its ends are equal?

$$\begin{array}{r}
 2.0833 \text{ one quarter of the circumf.} \\
 2.0833 \\
 \hline
 62499 \\
 62499 \\
 166604 \\
 416660 \\
 \hline
 4.34013889 \\
 15 \text{ Length.} \\
 \hline
 2170069445 \\
 434013889 \\
 \hline
 65.10208335
 \end{array}$$

Anf. 65 feet. 1 inch, and 2 tenths of an inch.

As the above operation is composed of a great number of figures, the major part of which are of no real use, the work may be much shortened by inverting the multiplicator, as explained in page 25 of the introduction to this treatise.

Discip. I perceive there is no difficulty in finding the solidity of any piece of round timber, provided the dimensions are given; but I should be glad to know, what method is taken by practical measurers to find the circumference.

Philo. The method is very easy, being nothing more than this: They girt the piece round with a string, double the string twice, and measure it on a rule, by which means they obtain one quarter of the girt or circumference, which they consider as the side of a square, whose area is equal to the area of the circular base.

Discip. But is this method true?

Philo. No: it is very erroneous; for the fourth part of the circumference of a circle, is not equal to the side of a square, whose area is equal to the area of the circle. For if the circumference of a circle be 1, the side of a square of equal area to that circle, is 0.2821, (see page 29) whereas by the false method of the girt it is only 0.25.

Discip. Why then is it practised? why is not the true method used instead of the false?

Philo. It was first introduced by ignorance, and continued, because it is somewhat easier than the other, till custom fixed it so firmly, that all the demonstrations in the world will not be able to banish it from general practice. But the solidity of round timber may be found very near the truth, by the following

R U L E.

Multiply the square of the circumference of the piece by the length, and that product by 0,07958; the last result will be the solidity required.

E X A M P L E I.

What is the solidity of a piece of timber, the fourth of whose circumference is 1 foot 6 inches, and its length 20 feet?

Solu-

Solution by Decimals:

$$\begin{array}{r}
 6 \text{ Circumference.} \\
 \underline{6} \\
 36 \\
 20 \text{ Length.} \\
 \hline
 720 \\
 .07958 \\
 \hline
 5760 \\
 3600 \\
 6480 \\
 5040 \\
 \hline
 57.29760 \\
 12 \\
 \hline
 3.5712 \\
 12 \\
 \hline
 6.8544
 \end{array}$$

Anf. 57 feet, 3 inches, 6 parts.

Solution according to the common Method.

$$\begin{array}{r}
 1.5 \text{ one fourth of the girt.} \\
 \sim \underline{1.5} \\
 75 \\
 15 \\
 \hline
 2.25 \\
 20 \text{ Length.} \\
 \hline
 45.00
 \end{array}$$

Anf. 45 feet.

Hence it appears, that 12 feet, $3\frac{1}{2}$ inches of timber are lost to the seller by using the common method.

EXAMPLE II.

Required the solidity of a piece of timber, when the fourth of the girt is 2 feet, and the length 30 feet.

Solution by the true method.

$$\begin{array}{r}
 8 \text{ Circumference.} \\
 \underline{8} \\
 64 \\
 30 \text{ Length.} \\
 \underline{1920} \\
 .07958 \\
 \underline{15360} \\
 9600 \\
 17280 \\
 \underline{13440} \\
 152.79360
 \end{array}$$

Ans. 152.7936 feet, or 152 feet 9 inches and a half.

Solution by the common Method.

$$\begin{array}{r}
 2 \\
 \underline{2} \\
 4 \\
 \underline{30} \\
 120 \text{ Feet, the answer.}
 \end{array}$$

The difference between these two solutions is 32 feet 9 inches and a half.

I shall add, my dear Discipulus, a solution to this question, by the accurate method laid down in page 134, for finding the solid content of a cylinder.

The diameter of a circle, whose circumference is 8, will be 2.546, this being squared is 6.482116; which multiplied by .7854, (see page 28) is 5.0910429064, the area of the base; this area multiplied by the length gives the solidity required.

OPERATION.

OPERATION.

$$\begin{array}{r}
 2.546 \\
 2.546 \\
 \hline
 15276 \\
 10184 \\
 12730 \\
 5092 \\
 \hline
 6.482116 \\
 .7854 \\
 \hline
 25928464 \\
 32410580 \\
 51856828 \\
 \hline
 45374712 \\
 5.0910429064 \\
 \hline
 30 \text{ Length.} \\
 \hline
 152.7312871920
 \end{array}$$

Ans. 152.73, &c. feet, not differing from the first solution one tenth of an inch; a sufficient proof that the above rule is sufficiently near the truth for common practice.

2. Of measuring tapering timber.

Discip. What am I to understand by tapering timber?

Philo. By tapering timber is meant, trees, &c. as they grow in the forests, or any pieces of timber, that are larger at one end than the other.

Discip. What method must be pursued in order to find the solid content of such pieces of timber?

Philo. The method generally used by measurers is very easy and expeditious; it is this: They measure along the piece from one end to the other, for the length; and at the middle, they girt it round with a string, double that string twice, and measure the length of it thus doubled, and call this the fourth of the girt, and suppose it equal to the side of the square, which the piece will hold throughout. In consequence of this, they square the fourth part of the girt, that is,

N 3

they

they multiply it by itself; and that product multiplied by the length of the piece, gives the content.

But if the tree has its bark on, they make an allowance for it; deducting it from the fourth part of the girt before they multiply it by itself.

Discip. What are the general allowances made for the bark?

Philo. There is not any fixed proportion for this allowance, that is general in all places; but the following are commonly used. For oak one tenth, or one twelfth part of the whole girt is deducted for the bark; in elm, beech, ash, &c. it is something less. Some measurers insist upon one inch, out of the fourth part of the girt. That is, if the quarter of the girt be 10 inches, they call it nine, and with that proceed to measure the tree. When the allowance for the bark is settled, the tree is measured by the following

R U L E.

From the girt, or quarter of the girt, according to the nature and thickness of the bark, deduct the allowance, and with the remainder find the content of the tree, as mentioned above.

E X A M P L E I.

Suppose it were required to find the content of an old thick-barked tree, whose length is 22 feet, and the quarter of the girt 2 feet 10 inches; the allowance for the bark being 1 inch out of the fourth of the girt.

Operation

Operation by Decimals.

$$\begin{array}{r}
 2.8333 \\
 \underline{.0833} \text{ Allowance for bark.} \\
 2.7500 \\
 \underline{2.75} \\
 1375 \\
 1925 \\
 \underline{550} \\
 7.5625 \\
 \underline{22} \text{ Length.} \\
 151250 \\
 \underline{151250} \\
 166.3750 \\
 \underline{12} \\
 4.500
 \end{array}$$

Ans. 166 feet, $4\frac{1}{2}$ inches.

By cross Multiplication.

$$\begin{array}{r}
 2-10 \\
 \underline{1} \text{ Allowance for bark.} \\
 2-9 \\
 \underline{2-9} \\
 2-0-9 \\
 \underline{5-6} \\
 7-6-9 \\
 \underline{22} \text{ Length.} \\
 26-4-6 \\
 \underline{14} \\
 166-4 \quad 6
 \end{array}$$

Ans. 166 feet, 4 inches, and 6 twelfths, or $4\frac{1}{2}$ inches, the same as before.

EXAMPLE II.

What is the content of a tree, whose length is 25 feet, and the quarter of the girt 3 feet 10 inches, the

N 4

allow-

allowance for bark being one twelfth of the circumference.

Operation by Decimals.

$$\begin{array}{r}
 12 \overline{) 3.8333 | .3194} \\
 \underline{3.6} \\
 23 \\
 \underline{12} \\
 113 \\
 \underline{108} \\
 53 \\
 \underline{48} \\
 .5
 \end{array}$$

3.8333

3194

3.5139

3.5139 reduced quarter of the girt.

316251

105417

35139

175695

105417

12.34749321

25 Length.

6173746605

2469498642

308.68733025

12

8.244

12

2.928

Ans. 308 feet, 8 inches, 2 parts and nine tenths.

By

By cross Multiplication.

$$\begin{array}{r}
 12 \overline{) 3-10} \\
 \underline{0-3-10} \\
 \text{From } 3-10 \\
 \text{Take } \underline{3-10} \\
 \text{Rem. } 3-6-2 \text{ Reduced } \frac{1}{4} \text{ girt.} \\
 \underline{3-6-2} \\
 7-0-4 \\
 1-9-1-0 \\
 10-6-6 \\
 \underline{12-4-2-0} \\
 25 \text{ Length.} \\
 68-8-2 \\
 \underline{24} \\
 308-8-2
 \end{array}$$

Ans. 308 feet, 8 inches, 2 twelfths; nearly the same as before.

EXAMPLE III.

A timber merchant brought three pieces of timber of the following dimensions, after the bark is allowed for, viz. No. I. 20 feet long, and 2 feet 3 inches the quarter of the girt; No. II. 30 feet long, and 1 foot 9 inches the quarter of the girt; and No. III. 27 feet long, and 2 feet 6 inches the quarter of the girt, at 1s. per foot; what will the whole amount to?

Operation by Decimals.

NUMB. I.

$$\begin{array}{r}
 2.25 \\
 2.25 \\
 \hline
 1125 \\
 450 \\
 \hline
 450 \\
 \hline
 5.0625 \\
 20 \\
 \hline
 F. 101.2500 \\
 12 \\
 \hline
 I. 3.00 \\
 \hline
 \end{array}$$

NUMB. II.

$$\begin{array}{r}
 1.75 \\
 1.75 \\
 \hline
 875 \\
 1225 \\
 \hline
 175 \\
 \hline
 3.0625 \\
 30 \\
 \hline
 F. 91.8750 \\
 12 \\
 \hline
 I. 10.500 \\
 \hline
 \end{array}$$

NUMB. III.

$$\begin{array}{r}
 2.5 \\
 2.5 \\
 \hline
 125 \\
 50 \\
 \hline
 6.25 \\
 27 \\
 \hline
 4375 \\
 1250 \\
 \hline
 F. 168.75 \\
 12 \\
 \hline
 I. 9.00 \\
 \hline
 \end{array}$$

F.I.

F. I.

No. 1. — 101.25
 2. — 91.875
 3. — 168.75
361.875

As 1 foot : 12. (= .05) :: 361.875
.05

£. 18.09375

20

s. 1.87500

12

d. 10.500

4

f. 2.0

Ans. The content of the whole is 361, feet, 10½ inches; and the price 18l. 1s. 10d. ½.

By Cross Multiplication.

NUMB. I.

2--3 Quarter girt.

2--3

6--9

4--6

5--0--9

20 Length.

101--3--0

NUMB. II.

1--9 Quarter girt.

1--9

1--3--9

1--9

3--0--9

30 Length.

91--10--6

N 6

NUMB.

THE WHOLE ART

NUMB. III.

2--6 Quarter girt.

$$\begin{array}{r}
 2--6 \\
 \hline
 1--3--0 \\
 5--0 \\
 \hline
 6--3--0 \\
 27 \\
 \hline
 48--9 \\
 12 \\
 \hline
 168--9
 \end{array}$$

	F.	I.
No. 1.	101	3
2.	91	10 $\frac{1}{2}$
3.	168	9
	361	10 $\frac{1}{2}$

Ans. 361 feet 10 $\frac{1}{2}$ inches; the same as before.

EXAMPLE IV.

A merchant has bought a fall of timber, consisting of ten trees, whose dimensions, when the bark is allowed for, are as follows:

	F. I.	
No. 1.	0--9	Length. 10
2.	0-10 $\frac{1}{2}$	14
3.	1--3	11
4.	1--8 $\frac{1}{2}$	16
5.	1--4	24
6.	1--4	23
7.	3--0	18
8.	2--2	28
9.	2--7	17
10.	2--11	30

What will the whole amount to at 9d. $\frac{1}{2}$ per foot?

Operation

Operation by Decimals:

NUMB. I.

.75 Quarter girt.
.75
 375
525
 5625

10 Length.

5.6250 Content.

NUMB. II.

.875 Quarter girt.
.875
 4375
 6125
7000
 765625

14 Length.

3062500
765625
 10.718750 Content.

NUMB. III.

1.25 Quarter girt.
1.25
 625
 250
125
 1.5625

11 Length.

17.1875 Content.

NUMB.

THE WHOLE ART

NUMB. IV.

1.708 Quarter girt.

1.708

13664

119560

1708

2.917264

16 Length.

18503584

291726446.676224 Content.

NUMB. V.

1.3333 Quarter girt.

1.3333

39999

39999

39999

39999

13333

1.77768789

24 Length.

711075156

35553757842.66450736 Content.

NUMB. VI.

1.3333 Quarter girt.

3333.1 Multiplier inverted.

13333

3999

399

39

3

1.7776

23 Length.

53328

3555240.8848 Content.

NUMB.

NUMB. VII.

3 Quarter girt.

3

9

18 Length.

162 Content;

NUMB. VIII.

2.16666 Quarter girt.

66661.2 Multiplier inverted.

433333

21666

12099

1299

129

12

4.69131

28 Length.

3753048

938262

131.35668 Content.

NUMB. IX.

2.58333 Quarter girt.

33385.2 Multiplier inverted.

516666

129166

20666

774

75

7

6.67357

17 Length.

4671499

667357

113.45069

NUMB.

THE WHOLE ART

NUMB. X.

2.91666 Quarter girt.

66619.2 Multiplier inverted.

583333

262499

2916

1749

174

17

8.50792

30 Length:

255.23760 Content.

No. 1. — 5.625

2. — 10.71875

3. — 17.1875

4. — 46.67622

5. — 42.6945

6. — 40.8848

7. — 162.

8. — 131.3568

9. — 113.45069

10. — 255.2376

825.79186

As 1 foot : to 9d. $\frac{1}{2}$ (= .03958) : : 825.7918385930 inverted.

24773

7432

412

65

£. 32.684

20

s. 13.680

12

d. 8.16

Ans. 32l. 13s. 8d.

By

By cross Multiplication.

NUMB. I.

0--9 Quarter girt.

$$\begin{array}{r}
 9--9 \\
 \hline
 6--9 \\
 10 \text{ Length.}
 \end{array}$$

$$\begin{array}{r}
 5--7--6 \text{ Content.} \\
 \hline
 \end{array}$$

NUMB. II.

0-10--6 Quarter girt.

$$\begin{array}{r}
 0-10--6 \\
 \hline
 0--5--3--0 \\
 8--9--0 \\
 \hline
 9--2--3 \\
 14 \text{ Length.}
 \end{array}$$

$$\begin{array}{r}
 10--8--7--6 \text{ Content.} \\
 \hline
 \end{array}$$

NUMB. III.

1--3 Quarter girt.

$$\begin{array}{r}
 1--3 \\
 \hline
 3--9 \\
 1--3 \\
 \hline
 1--6--9 \\
 11 \text{ Length.}
 \end{array}$$

$$\begin{array}{r}
 17--2--3 \\
 \hline
 \end{array}$$

NUMB. IV.

1--8--6 Quarter girt.

$$\begin{array}{r}
 1--8--6 \\
 \hline
 10-3--0 \\
 1--1--8--0 \\
 1--8--6 \\
 \hline
 2-11--0--3--0 \\
 16 \text{ Length.}
 \end{array}$$

$$\begin{array}{r}
 46--8--4--0 \text{ Content.} \\
 \hline
 \end{array}$$

NUMB.

THE WHOLE ART

NUMB. V.

1--4 Quarter girt.

1--4

5--4

1--4

1--9--4

24 Length.42--8--0 Content.

NUMB. VI.

1--4 Quarter girt.

1--4

5--4

1--4

1--9--4

23 Length.40--10--8 Content.

NUMB. VII.

3--0 Quarter girt.

3--0

9

18 Length.162 Content.

NUMB. VIII.

2--2

2--2

4--4

4--4

4--8--428 Length.131--5--4 Content.

NUMB.

NUMB. IX.

2--7 Quarter girt.

$$\begin{array}{r}
 2--7 \\
 \hline
 1--6--1 \\
 5--2 \\
 \hline
 6--8--1 \\
 \hline
 17 \\
 \hline
 113--5--5 \text{ Content.}
 \end{array}$$

NUMB. X.

2--11 Quarter girt.

$$\begin{array}{r}
 2--11 \\
 \hline
 2--8--1 \\
 5--10 \\
 \hline
 8--6--1 \\
 \hline
 30 \text{ Length.} \\
 \hline
 255--2--6 \text{ Content.}
 \end{array}$$

No. 1.	5	7	6
2.	10	8	7--6
3.	17	2	3
4.	46	8	4
5.	42	8	
6.	40	10	8
7.	162.		
8.	131	5	4
9.	113	5	5
10.	255	2	6
Total	825	10	7--6

Such is the method generally used in measuring timber; but in order to prevent mistakes, several authors have calculated tables, whereby the content of any piece of timber is easily known, by having the length and the quarter of the girt given. Nor is it at all difficult, though something tedious to calculate such tables for your own use.

Discip. I should be very glad to know the method taken in forming tables of this kind: I have seen tables of various kinds, in books I have occasionally read on other subjects; but always thought they could not

not be composed without the utmost labour and difficulty.

Philo. Some tables are indeed sufficiently difficult and tedious; but the generality, especially those for measuring timber, are easily computed. I will give you an instance or two of this, by computing part of a table of that kind. But it will be previously necessary to observe, that a table sufficient to answer the purposes of measuring timber, must be computed from six inches to 54 inches the side of the square, and from one fourth of a foot to 45 feet in length.

This being premised, let it be required to compute a table for measuring a piece of timber, whose side of the square is seven inches, but of any length less than 26 feet.

In order to find the content of one foot in length, multiply the seven inches by seven inches, and the result will be four inches and one twelfth, thus :

$$\begin{array}{r} 7--0 \\ 7--0 \\ \hline 4--1 \end{array}$$

This will be the first line in the table; the second line will be the first doubled, or added to itself; the third the first and second added together; the fourth, the third, and first added together: and thus, by a continual addition of the first line, the table will be readily constructed without any farther difficulty. See the whole work.

F. long. F. I. P.

1	0	4	1
		4	1
2	0	8	2
		4	1
3	1	0	3
		4	1
4	1	4	4
		4	1
5	1	8	5
		4	1

F. long

F. long. F. I. P.

6	2	0	6
		4	1
7	2	4	7
		4	1
8	2	8	8
		4	1
9	3	0	9

It will, perhaps, be unnecessary to observe, that the figures 4--1, that stand between the lines are to be omitted, when the table is finished; they are only placed there to shew the method of computing the contents that are set against their respective lengths. When they are omitted, the numbers will stand as below.

A Table shewing the solid content of any piece of timber, seven inches the side of the square, and not exceeding 26 feet in length.

F. long. F. I. P.

1	0	4	1
2	0	8	2
3	1	0	3
4	1	4	4
5	1	8	5
6	2	0	6
7	2	4	7
8	2	8	8
9	3	0	9
10	3	4	10
11	3	8	11
12	4	1	0
13	4	5	1
14	4	9	2
15	5	1	3
16	5	5	4
17	5	9	5
18	6	1	6
19	6	5	7
20	6	9	8

F. long.

THE WHOLE ART

F. long. F. I. P.

21	7	0	1	9
22	7	5	10	
23	7	9	11	
24	8	2	0	
25	8	6	1	
26	8	8	2	

If you calculate a set of tables in this manner, for every quarter of an inch the side of the square, and extend the length to 45 feet, you will have a very useful compendium for measuring timber with accuracy and dispatch. There are indeed several books extant filled with tables of this kind: But I would not have you depend implicitly upon any tables, till you have thoroughly examined and corrected them; as there are errors even in the best.

With regard to the quarter, half, or three quarters of a foot, happening often in the length, it will be sufficient to calculate the contents of these, and set them down at the bottom of the table. These contents are easily found, if you consider that the one quarter of a foot is one fourth of the first line, &c. and therefore the contents of these fractions are:

F. I. P. S.

 $\frac{1}{4}$ 0--1--0--3. $\frac{1}{2}$ 0--2--0--6. $\frac{3}{4}$ 0--3--0--9.

These contents, when they happen in the length, are to be added to the contents found in the table, and the sum will be the area required.

EXAMPLE.

What is the solid content of a piece of timber, whose side of the square, or quarter girt, is seven inches, and the length 23 feet nine inches, or 23 feet and three quarters.

The content of 23 feet

F. I. P.

7--9--11

0--3--0--9

8--0--11--9

Ans.

OF MEASURING.

235

Ans. 8 feet, 0 inches, 11 parts, 9 seconds.

In order to render the method of calculating, and consequently of examining tables as easy as possible, I shall give another instance, where the first line has second and third parts in it, as the method of proceeding is something different from that of the former.

Let it be required to compute a table for measuring timber, whose quarter girt, or side of the square, is eight inches and a quarter.

	F. long.	F. I.	P.	S.	T.
8--3	1	0	5	8	0 9
<u>8--3</u>		0	5	8	0 9
2--0--9	2	0	11	4	1 6
<u>5--6--0</u>		0	5	8	0 9
5--8--0--9 first line.	3	1	5	0	2 3
		0	5	8	0 9
	4	1	10	8	3 0
		0	5	8	0 9
	5	2	4	4	3 9
		0	5	8	0 9
	6	2	10	0	4 6
		0	5	8	0 9
	7	3	3	8	5 3
		0	5	8	0 9
	8	3	9	4	6 0
		0	5	8	0 9
	9	4	3	0	6 9

In this manner the above table is computed, and when the intermediate lines, together with the second and third parts are omitted, the contents will stand in the following manner:

F. long

THE WHOLE ART

F. long.	F.	I.	P.
1	0	5	8
2	0	11	4
3	1	5	0
4	1	10	8
5	2	4	4
6	2	10	0
7	3	3	8
8	3	9	4
9	4	3	0
10	4	8	8
11	5	2	4
12	5	8	0
13	6	1	8
14	6	7	4
15	7	1	0
16	7	6	9
17	8	0	5
18	8	6	1
19	8	11	9
20	9	5	5
21	9	11	1
22	10	4	9
23	10	10	5
24	11	4	1
25	11	9	9
26	12	3	5
27	12	9	1
28	13	2	9
29	13	8	5

The Quarters are:

	F.	I.	P.	S.
$\frac{1}{4}$	0	1	5	0
$\frac{1}{2}$	0	2	10	0
$\frac{3}{4}$	0	4	3	0

By this table you may very readily find the solid content of any piece of timber, whose side of the square, or quarter girt, is $8\frac{1}{4}$ inches, and the length not exceeding 29 feet.

EXAMPLE.

What is the solid content of a piece of timber, whose quarter girt is $8\frac{1}{4}$ inches, and its length 24 feet and a half?

	F.	I.	P.
Content of 24 feet.	11	4	1
of $\frac{1}{2}$	0	2	10
	11	6	11

Ans. 11 feet, 6 inches, 11 parts, or $\frac{3}{4}$ of an inch.

Discip. I was unwilling to interrupt you, till you had finished the solution, but should now be glad to know why the solid content of 3, 6, and 9 inches only, are computed, and all the intermediate inches omitted.

Philo. The reason for their being omitted is, the small increase they would add to the content of the piece of timber: Measurers, indeed, seldom take notice of any other denominations in the length of a piece, than feet and quarters. But this is a defect that should be supplied in large pieces of timber, especially when the wood is of a considerable price, such as mahogany, &c. And I would recommend to you, in forming a set of tables, to remember, when the side of the square is more than 12 inches, or one foot, to place at the bottom, the contents of the several inches; it will only add nine lines to each table, and, by that small increase, you may compute the solidity to sufficient accuracy. Let us, for instance, see the difference between the contents of a large piece of timber, when the intermediate inches are taken in, and when omitted in the computation.

EXAMPLE.

What will be the contents of two pieces of timber, one 3 feet 4 inches the side of the square, and 20 feet 6 inches long; and the other 3 feet 4 inches the side of the square, and 20 feet 8 inches long?

OPERATION.

First piece 3—4 Side of the square.

$$\begin{array}{r} 3—4 \\ 1—1—4 \\ 10—0 \end{array}$$

11—1—4 Area of the base.

20—6 Length.

$$\begin{array}{r} 5—6—8—0 \\ 221—11—8 \\ 227—6—4—0 \end{array}$$

Second piece 3—4

$$\begin{array}{r} 3—4 \\ 1—1—4 \\ 10—0 \end{array}$$

11—1—4

20—8 Length.

$$\begin{array}{r} 7—4—10—8 \\ 221—11—4 \\ 229—4—2—8 \end{array}$$

	F.	I.	P.	S.
The content of the 2d is	229	4	2	8
Ditto ——— of the 1st	227	6	4	0
	..	1	9	10—8

The difference is 1 foot, 9 inches, 10 parts, 8 second parts.

If you are desirous of computing decimal tables, it may be easily done; and they will have this advantage, that the content may be found to any degree of accuracy you may think necessary. One instance will be sufficient to shew the method of computing decimal tables for measuring timber.

EXAMPLE.

Let it be required to compute a decimal table, for finding the solid content of any piece of timber, whose side of the square is 18 inches, or 1 foot 6 inches?

OF MEASURING.

239

1.5 Side of the square.

$$\begin{array}{r} 1.5 \\ \hline 75 \\ \hline 15 \end{array}$$

2.25 First line.

1	2.25
	<u>2.25</u>
2	4.20
	<u>2.25</u>
3	6.75
	<u>2.25</u>
4	9.00
	<u>2.25</u>
5	12.25
	<u>2.25</u>
6	14.50

In this manner the table may be easily constructed, and the numbers will stand as in the margin.

1	2.25
2	4.5
3	6.75
4	9.
5	11.25
6	13.5
7	15.75
8	18
9	20.5
10	22.25
11	24.75
12	27
13	29.5
14	31.25
15	33.75
16	36
17	38.5
18	40.25
19	42.75
20	45
21	47.5
22	49.25
23	51.75
24	54
25	56.5
26	58.25
27	60.75
28	63
29	65.5
30	67.25
31	69.75
32	72

Oz

The

The use of this table is the same with the former, only the content here is given in feet and decimals, instead of feet, inches, and parts.

EXAMPLE.

What is the content of a piece of timber, whose side of the square is 1 foot 6 inches, and the length 19 feet.

Ans. 42.75, or 42 feet, 9 inches.

What has been observed will be abundantly sufficient to explain the nature and use of these tables; and I shall only observe, with regard to their construction, that the lines forming a series of numbers in arithmetical progression, whose difference is the first line, there will often be no occasion for the continued addition; but the numbers may be set down with the greatest facility, without any calculation. The decimal table affords us a plain example of this kind, for the decimals are nothing more than an arithmetical progression, increasing by .25, or one quarter of a foot. The series therefore is .25, .50 (or .5) .75, 1.0. And these will be continually repeated from the beginning of the table to the end, be its length what it will. The line of feet is also a series of numbers in arithmetical progression, whose common difference is 2, except where there is only a cypher, or blank, in the decimal series, when the common difference will be 3. Consequently the numbers are only 2, 4, 6, 8, 10, 12, 14, 16, 18, and may be readily set down without any farther trouble, than that of adding three instead of two to every fourth number.

Discip. I am exceedingly obliged to you for the trouble you have taken in explaining these useful parts of mensuration in so conspicuous a manner, but should be glad to know the best method of finding the amount of any piece of timber, when the content is found by cross multiplication. You have given a sufficient number of instances by decimals; but as the other is often practised, and, I think, in some instances, with considerable advantage, I should often

often use it, provided I knew how to find the value, or what the piece will amount to at a certain rate per foot.

Philo. I am always pleased to satisfy any doubts that may perplex your mind, or remove any difficulties, that may obstruct your progress. I intended to give one instance of this kind before I finished this branch of mensuration: but as I perceive it is a difficulty of more consequence than I imagined, and that the removing this difficulty will give you pleasure, I shall be very explicit in this particular, and do not doubt of giving you full satisfaction.

In order to this, it will be necessary to consider the nature of the fractions into which the foot, &c. is divided; namely, twelfths. That is, the foot is divided into twelve equal parts called inches; the inches into twelve equal parts called the twelfth of an inch, or simply, parts; and these are again divided into other twelve equal parts called seconds, &c. Consequently the inches are twelfths of a foot, every part the 144th of a foot ($= 12 \times 12$) and every second the 1728th part ($12 \times 12 \times 12$).

The respective value of these divisions would be very readily found, if the denominations of money decreased in the same proportion: But this is not the case; there is only one denomination that has this proportion, viz. Shillings and pence. When, therefore, the price of timber is 1s. per foot, every inch will be a penny, and every part the twelfth of a penny, or one third of a farthing; consequently three of these parts are equal to a farthing. Hence it follows, that when the price of timber is any number of shillings per foot, the value is also very easily found; for if the whole content be multiplied by the number of shillings, the result will be the number of shillings. But when this does not happen, the value of the whole must be found, by taking the aliquot parts. A few examples will render the whole very intelligible.

THE WHOLE ART

EXAMPLE I.

What will a piece of timber, 1 foot, 6 inches the side of the square, and 20 feet long, amount to at 1s. per foot?

OPERATION.

$$\begin{array}{r}
 1-6 \\
 1-6 \\
 \hline
 9-0 \\
 1-6 \\
 \hline
 2-3-0 \\
 20 \\
 \hline
 45-0-0 \text{ Content.}
 \end{array}$$

Ans. 45 shillings, or 2l. 5s.

EXAMPLE II.

What will be the value of a piece of timber 25 feet long, and 1 foot 4 inches the side of the square, at 1s. per foot?

OPERATION.

$$\begin{array}{r}
 1-4 \\
 1-4 \\
 \hline
 5-4 \\
 1-4 \\
 \hline
 1-9-4 \\
 25 \\
 \hline
 44-5-4
 \end{array}$$

	£.	s.	d.
The 44 feet	2	4	0
The 5 inches	0	0	5
The 4 twelfths	0	0	0 $\frac{1}{4}$
	2	4	5 $\frac{1}{4}$

Ans. 2l. 4s. 5d. $\frac{1}{4}$.

EXAMPLE

EXAMPLE III.

A gentleman sold three pieces of timber of the following dimensions, viz.

F. 1.

No. 1. ——— 2—5 Side of the square 25 feet long.

2. ——— 1—9 27

3. ——— 1—10 40

What will the whole amount to at 2s. per foot?

OPERATION.

	F.	I.	P.
Content of No. 1. ———	146	c	1
2. ———	82	8	3
3. ———	134	5	4
Whole content	363	1	8
	£.	s.	d.
The 363 feet ———	36	6	0
The 1 inch ———	0	0	2
The 8 twelfths ———	0	0	1½
	£36	6	3½

Ans. 36*l.* 6*s.* 3*d.* ½.

EXAMPLE IV.

What must a timber-merchant pay for three pieces of timber, of the following dimensions, viz.

F. S.

No. 1. ——— 2 1 Side square 20 feet long.

2. ——— 3 2 30

3. ——— 1—11 27

at 1*s.* 6*d.* per foot?

Content of No. 1. ———	86	9	8.
2. ———	360	10	0
3. ———	99	2	3
Content of the whole	486	9	11

The 486 feet at 1s. per foot is	24—6—0
Ditto, at 6d.	12—3—0
The 9 inches at 1s. per foot	0—0—9
Ditto, at 6d.	0—0—4½
The 11 twelfths	0—0—1½
	£ 36—10—3

Ans. 36l. 10s. 3d.

I have been so very explicit in these solutions, that I think any more will be unnecessary, as the manner of proceeding in all is the same, the different price per foot making the only difference in the operation.

Discip. I am extremely obliged to you, Sir; you have removed all my difficulties; I can now find the value as easily as the content of any piece of timber.

Of measuring stone, &c.

Philos. Though I have thought proper to divide the mensuration of stone, &c. from that of timber, yet they are actually the same, and performed by the same rules, except in taking the dimensions, which is done with more accuracy in stone, because the value is generally much greater.

Discip. What are the methods used in taking the dimensions of stone.

Philos. They measure the length, breadth, and thickness with a rule, but with such accuracy, that even the twelfth of an inch is set down, in measuring a block of marble: in common stone they are not so accurate. When the dimensions are taken, the solidity is found as before by the following

R U L E.

Multiply the breadth into the depth or thickness, and that product by the length; the result is the solidity required.

E X A M P L E.

EXAMPLE I.

What is the solid Content of a piece of marble, whose breadth is 4 feet, 8 inches, and a half, its depth 3 feet 4 inches, and its length 8 feet.

Solution by Decimals.

$$\begin{array}{r}
 \text{Breadth } 4.7083 \\
 \text{Depth } \underline{3.3333} \\
 141249 \\
 141249 \\
 141249 \\
 141249 \\
 141249 \\
 \underline{141249} \\
 15.69417639 \\
 \underline{\hspace{1.5cm} 8 \text{ Length.}}
 \end{array}$$

$$\begin{array}{r}
 \text{F. } 125.55341112 \\
 \underline{\hspace{1.5cm} 12}
 \end{array}$$

$$\begin{array}{r}
 \text{I. } 6.6408 \\
 \underline{\hspace{1.5cm} 12}
 \end{array}$$

$$\begin{array}{r}
 \text{P. } 7.6896 \\
 \underline{\hspace{1.5cm} 12}
 \end{array}$$

$$\begin{array}{r}
 \text{S. } 8.2752 \\
 \underline{\hspace{1.5cm} 12}
 \end{array}$$

$$\text{Ans. } 125 \text{ F. } 6 \text{ I. } 7 \text{ P. } 8 \text{ S.}$$

By Cross Multiplication.

$$\begin{array}{r}
 4 \text{ --- } 8 \text{ --- } 6 \\
 \underline{\hspace{1.5cm} 3 \text{ --- } 4} \\
 1 \text{ --- } 6 \text{ --- } 10 \text{ --- } 0 \\
 14 \quad 1 \quad 6 \\
 \underline{\hspace{1.5cm} 15 \quad 8 \quad 4} \\
 \hspace{1.5cm} 8 \text{ Length.} \\
 \underline{\hspace{1.5cm} 125 \quad 6 \quad 8 \text{ Content.}}
 \end{array}$$

It will be needless to give any more instances of this kind, the operations being the very same with those already

already given for finding the solid content of square and unequal sided timber.

S E C T. II.

Of the best methods of taking the dimensions of timber, in order to avoid the gross errors that attend the common practice of measurers.

Discip. When you laid down the rule, page 214, I supposed, that if the method there pointed out was followed, the solution would be sufficiently accurate; can there then be any necessity for other rules and precepts?

Philo. The rule you mentioned is sufficiently accurate for finding the solidity of timber, whose ends are equal; but as this is rarely the case, other methods must be pursued, if the practitioner be desirous of avoiding the gross errors attending the common practice.

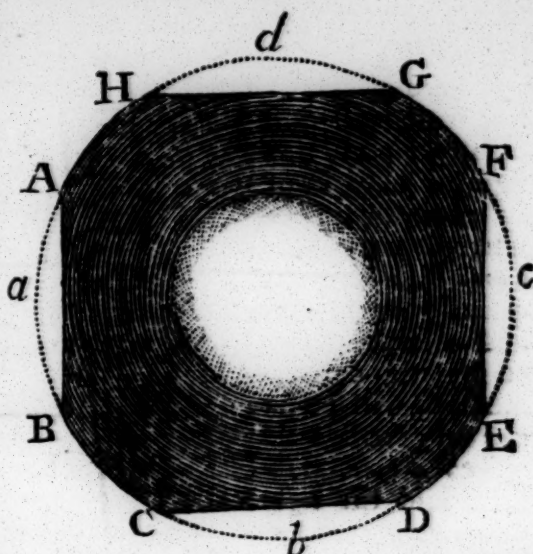
It is the custom in the King's yards, with regard to oak, elm, and beech, to have it squared by the merchant before it is received by the purveyors; and according to their contract, the sum of the breadth of the slabs taken off, are not to be less than twice the sum of the wanes; if they should be less the King's measurers hew the opposite sides till the dimensions are reduced to the terms of the contract; but they generally give the turn of advantage to the merchant. When the sides of the timber are thus square, the dimensions are taken with a pair of large callipers; and the different extents between the points of the instrument, being measured on a scale of inches, give the dimensions which they use instead of the girt.

Discip. In describing the method used for taking the dimensions of timber in the King's yards, you said, that the sum of the breadth of the *slabs* taken off, must not be less than twice the sum of the *wanes*. But as I have not an adequate idea of these terms, I should be glad if you would explain them to me.

Philo.

Philo. With all my heart Discipulus.

The figure I have drawn, will, I dare say, be abundantly sufficient for the purpose. Let therefore, the outer circle $A d G F e E D b C B a$ represent the end of a piece of timber; then will $B a A, H d G, F e E, D b C,$ repre-



sent the *slabs* that must be taken off, and $AH, GF, ED, CB,$ represent the *wanes*, or parts of the external circumference of the tree that are suffered to remain.

Now if the right lines $AB, CD, EF, GH,$ are when added together, equal to twice the sum of the lengths of the arches or wanes $AH, GF, ED, CB,$ the tree is properly prepared for the King's measurers; but if not, they hew the sides till they make them equal.

Discip. You have indeed removed the difficulty. I shall be at no loss in knowing when timber is, and when it is not properly prepared for being measured in the King's yards.

Philo. It gives me pleasure whenever my meaning is thoroughly understood; and I shall now proceed to lay down some useful rules, which you will do well to observe whenever you are called upon to measure timber.

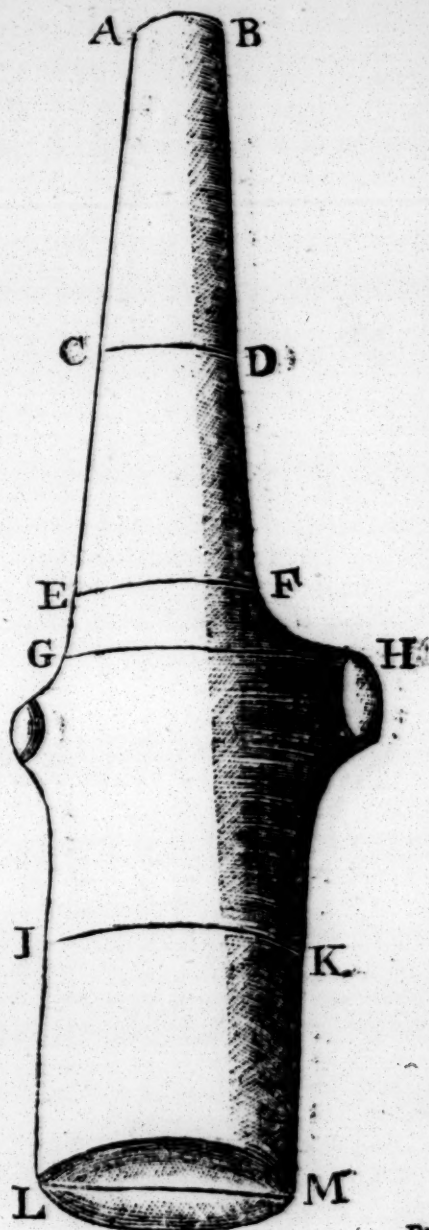
RULE I.

Gird the tree in as many places as you conveniently can, add these girts together, and divide the sum by their

their number; the quotient will be the mean girt of the tree.

The usefulness of this rule will sufficiently appear, if the figure I have drawn be attentively considered. It represents a tree in a form frequently met with, and which, if measured by the common method, will be sold at a very considerable loss to the seller. I have laid the figure down very accurately, from a scale, and the dimensions of it are these: the length is 31.7 feet, or 31 feet 8 inches, 4 parts, 3 seconds nearly. A quarter of the girt at $LM=IK,=GH$, is 1 foot 10 inches at EF , the middle of the tree, 1 foot 2 inches at CD , 1 foot 0, and at AB 7 inches.

Let it therefore be required to find the solid content of this tree, both by the preceding rule, and also by the common method of measuring.



By the preceding Rule.

One-fourth of the Girt at LM	F. I.
at IK	1—10
at GH	1—10
at EF	1—2
at CD	1—9
at AB	0—7
	<u>8—3</u>

6)8—3
1—4—6 Mean Quarter of the Girt=

Then 1.375

1.375

6875

9625

4125

1375

1.890625

31.7 Length.

13234375

1890625

5671875

59.9328125 Content.

F. 59.9328125

12

L. 11.193

12

2.316

Ans. 59 feet, 11 inches, 2 twelfths.

THE WHOLE ART

By the Common Method.

$$\begin{array}{r}
 1.1666 \text{ Quarter girt.} \\
 \underline{1.1666} \\
 69996 \\
 69996 \\
 69996 \\
 11666 \\
 \underline{11666} \\
 1.36095556 \\
 \underline{31.7 \text{ Length.}} \\
 952668892 \\
 136095556 \\
 \underline{408286668} \\
 \text{F. } 43.14 \text{ } 291152 \\
 \underline{12} \\
 \text{I. } 1.50
 \end{array}$$

Ans. 43 feet 1 inch and a half.

	F.	I.	P.
By the former method	59	11	2
By the common method	43	1	6
	<u>16</u>	<u>9</u>	<u>8</u>

Hence it appears, that the seller will lose 16 feet, 9 inches, 8 twelfths of timber, by selling it by the common method of measuring.

But what is, perhaps, still more surprising is, that if the tree were cut asunder at the line GH, the two pieces would measure more than when whole, even tho' the content were found by the first method.

Let us, for Example, suppose the tree cut asunder at the line GH, then will the length of the butt be 12 feet 6 inches, and the quarter of the girt 1 foot 10 inches. The other part will be 19 feet 2 inches and a half nearly, and the quarter of the girt at CD, now the middle of the piece, 1 foot.

F. I. P.

The solid content of the butt 42—0--2

Of the top, or upper part 19--2--4

61--2--6

By this method the solid content of the tree is 61 feet, 2 inches, 6 twelfths.

RULE II.

Never let the tops of your trees remain farther than they run about 7 or 8 inches the side of the square, for this will occasion the girting place nearer the top, and consequently make the product less; nor will the addition of the length make it sufficient to supply the deficiency of the girt.

To illustrate this Rule, I have drawn the figure in the margin, where the whole length of the tree is divided into six equal parts, the tree runs regularly tapering from the bottom to the top, and consequently the girts are in arithmetical progression. The length is 14 feet, and the quarters of the respective girts as follows:

That at the base

F. I. P.

is ————— 1--8--0

*f*D-1--5--6

*e*F-1--3--0

The middle *d*H-1--1--6

*c*K-0-11--0

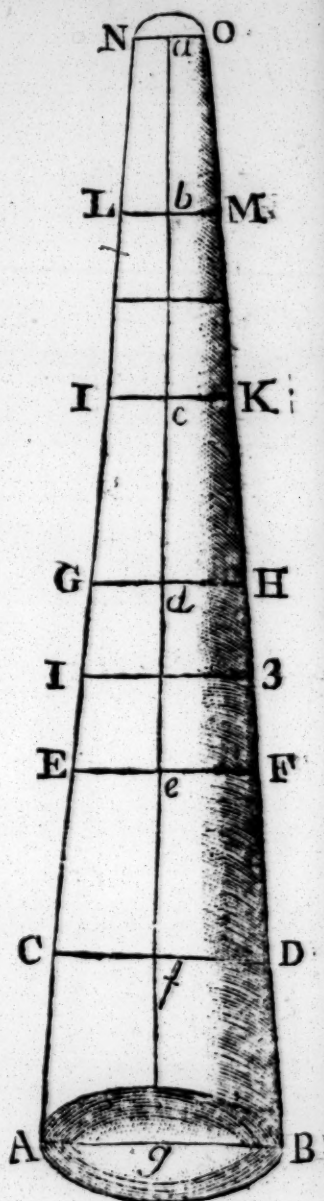
*l*M-0-8--6

*a*O-0-6--0

Suppose it were required to find the solid content of this piece of timber.

It will be 17 feet, 8 inches, 7 parts.

Let us now suppose the piece LMNO, 2 feet 4 inches in length, cut off, then will the length be 11 feet 8 inches, the middle of the piece, the line 1--3, whose half or quarter of the girt will be 1 foot 2 inches, 9 parts.



Then

Then will the content be

To which add the content of LMNO,

Content of the two pieces.

F. I.

17--4

1--0

18--4

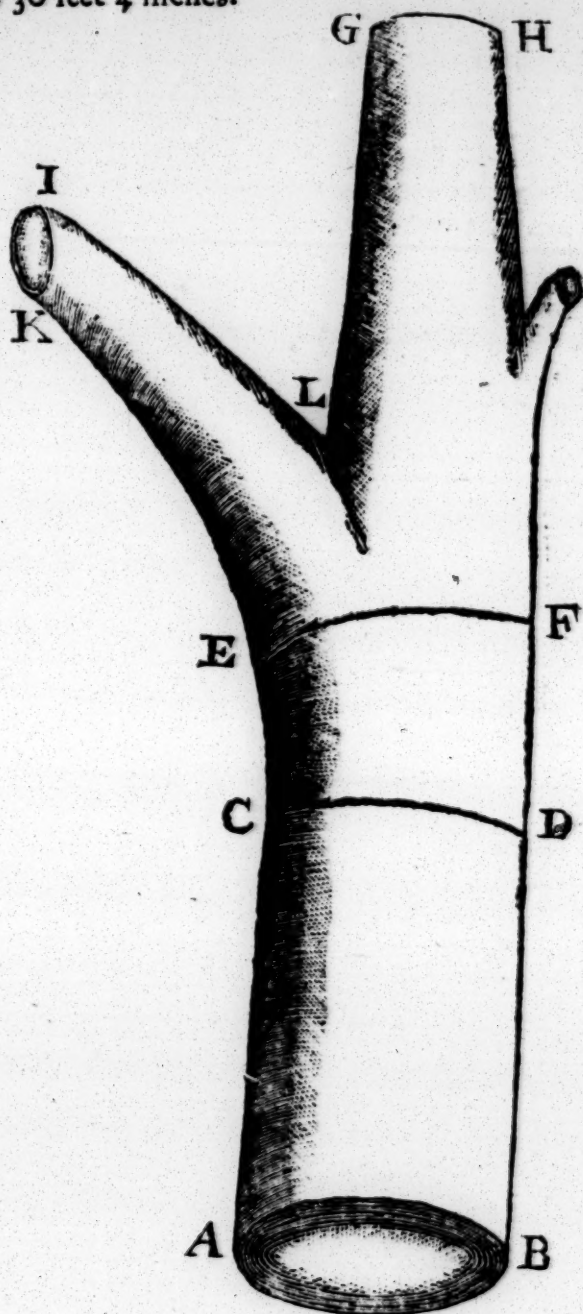
N. B. If the side of the square be less than six inches, it is not esteemed timber. But if any of the limbs are of that size, they are accounted timber measured as such, and their contents added to that of the body of the tree.

R U L E III.

It is a privilege enjoyed by the buyer, to take the girt in any part of the tree between the middle and the butt.

EXAMPLE.

Let the figure in the margin represent a tree, whose length is 30 feet 4 inches.



F. I.

The quarter at the butt AB 1--9

at CD 1--8

at EF 1--9

at GH 0-10

at IK 0-7

And the length of LM, of the }
branch IKS } 9-4

And let it be required to find the content of the whole.

As the quarter girt at EF, the middle of the tree, is greater than that of CD, the buyer will, doubtless, girt the tree there.

F. I.

Content of the tree, exclusive of the }
branch } 84--2

 of the branch

3--2

87--4

Ans. 87 feet, 4 inches.

But you should remember, that so large a branch growing in the manner represented in the figure, renders the stick of timber very valuable to Ship-wrights, as a useful piece, which they call a knee, may be cut out of this tree. This observation is worth regarding, for I have more than once seen very fine timber of this kind sold for much less than its real value, through the ignorance of the grower.

RULE IV.

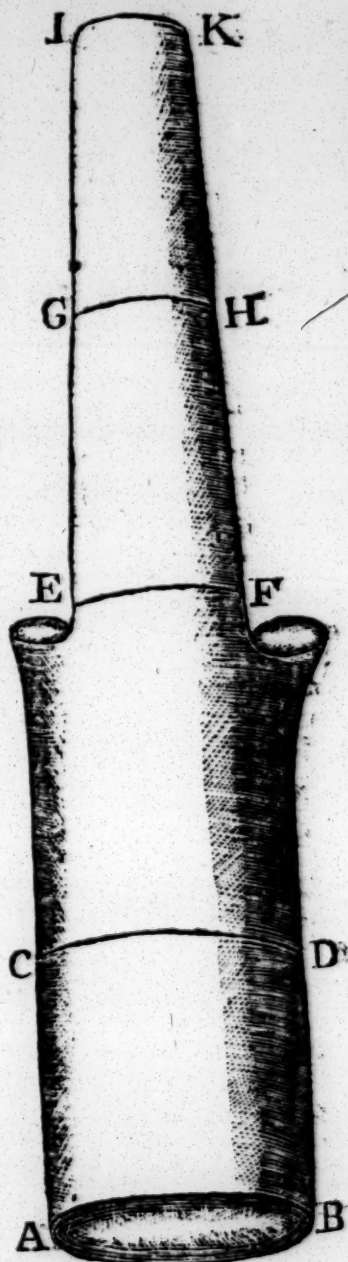
When a piece of timber does not run tapering with any regularity, it will be adviseable to cut it into two or three pieces, unless the buyer will consent to the measuring it in the manner it must be measured when divided into different pieces, and which practical measurers call *joggling* the piece.

EXAM-

EXAMPLE.

Let the figure in the margin represent a tree, whose length from the butt to the top is 12 feet, viz. from A to F 6 feet, and from F to I 6 feet; the quarter of the girt at the butt AB, and at CD, is 1 foot 6 inches; at EF, the middle of the piece 10 inches, at GH 8 inches and a half, and at IK 7 inches.

The very inspection of the figure, shews, that it would be very unjust to the feller, should the girt be taken at EF, the middle of the piece; and equally unjust to the buyer, if taken any where between the two knots at E and F. The best way therefore is to consider the stick of timber as composed of two pieces, AEFB, and EIKF. By this means justice will be done to both parties. Let us, therefore, see how many solid feet of timber it will contain, when properly measured.



First,

First, for the butt or lower part AEFB.

1--6 Quarter girt.

$$\begin{array}{r}
 1--6 \\
 \hline
 9--0 \\
 1--6 \\
 \hline
 2--3--0 \\
 \hline
 6 \text{ Length.} \\
 13--6--0
 \end{array}$$

Second, for the top part. EIKF.

$$\begin{array}{r}
 0--8 \\
 \hline
 0--8 \text{ Quarter girt.} \\
 \hline
 5--4 \\
 \hline
 6 \text{ Length.} \\
 \hline
 2--8--0 \text{ Content.} \\
 \hline
 \text{Butt } 13--6--0 \\
 \text{Top } 2--8--0 \\
 \hline
 16--2--0
 \end{array}$$

Had this piece of timber been measured at once, its content would have been as follows:

$$\begin{array}{r}
 0--10 \text{ Quarter girt.} \\
 \hline
 0--10 \\
 \hline
 8--4 \\
 \hline
 12 \text{ Length.} \\
 \hline
 8--4--0
 \end{array}$$

Content by two dimensions 16--2--0.

Ditto when measured at once 8--4--0

7--10--0

Consequently the feller would lose 7 feet 10 inches of timber by measuring it at once.

RULE V.

When the tree is crooked the length must not be taken by measuring either along the concave or convex side: but as near as you can along the middle; for the length taken on the convex side will be too great, and that measured on the concave too small.

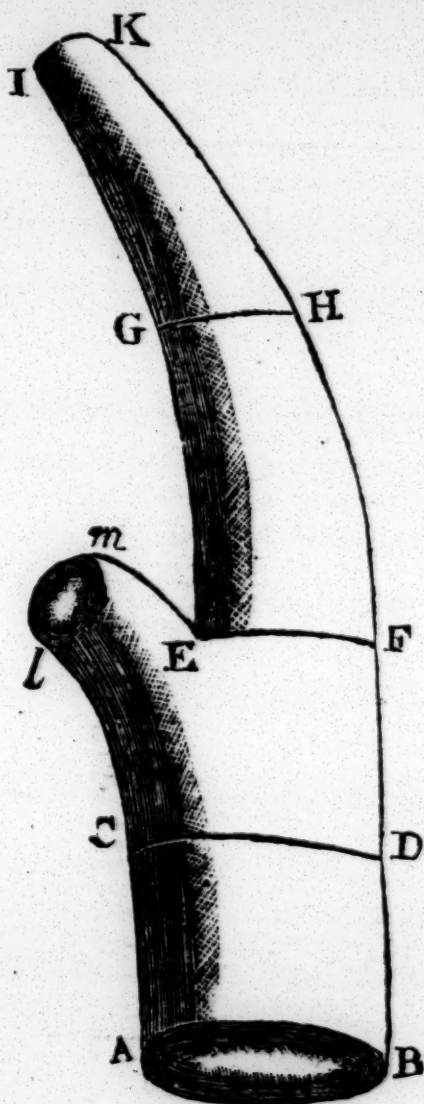
EXAMPLE

EXAMPLE.

Suppose a tree were to be measured, in the form of that in the figure below; the dimensions of which are as follows: the $\frac{1}{2}$ of the girt at AB and CD, 4 feet 1 inch; at EF, 2 feet 3 inches; at GH, 1 foot 9 inches; at IK 10 $\frac{1}{4}$ inches, and that of the branch at m , 1 foot 2 inches 1 twelfth: the length from B to F 7 feet, and from EF to IK, (measured along the middle of the piece) 7 feet, and of m F 2 feet 1 inch. What will be the content, and what will it amount to at 1 shilling and 9 pence per foot?

This

This piece being so very different in its dimensions, must be supposed to be cut asunder in the line EF, and the content of each piece added together; for that sum will be the content of the piece.



Solution

Solution by Cross Multiplication.

First for the butt, or lower end of the piece.

4--1 Quarter girt at CD.

$$\begin{array}{r}
 4--1 \\
 \hline
 4--1 \\
 16--4 \\
 \hline
 16--8--1 \\
 \hline
 7 \text{ Length.}
 \end{array}$$

116--8--7 Content.

Second, for the top, or upper part.

1--9 Quarter girt at GH.

$$\begin{array}{r}
 1--9 \\
 \hline
 1--3--9 \\
 1--9 \\
 \hline
 3--0--9 \\
 \hline
 7 \text{ Length.}
 \end{array}$$

21--5--3 Content.

Third, for the content of the branch *Im* E.

1--2--1 Quarter girt.

$$\begin{array}{r}
 1--2--1 \\
 \hline
 1--2--1 \\
 2--4--2 \\
 \hline
 1--2--1 \\
 \hline
 1--4--6--4--1 \\
 \hline
 2--1 \text{ Length.}
 \end{array}$$

$$\begin{array}{r}
 1--4--6--4--1 \\
 2--9--0--8--2 \\
 \hline
 2--10--5--2--6--1 \text{ Content.}
 \end{array}$$

F. I. P.

Content of the butt---	116	—	8	—	7
of the top---	21	—	5	—	3
of the branch	2---	10	—	5	
	141	—	0	—	3

141 Feet 1s. 9d. per foot is

1 Quarter or 3 twelfths of an inch is

$$\begin{array}{r} 12-6-9 \\ 0-0-0\frac{1}{4} \\ \hline \text{£. } 12-6-9\frac{1}{4} \end{array}$$

Ans. The content of the tree is 141 feet, 0 inches, 3 parts, and the amount 12l. 6s. 9d. $\frac{1}{4}$.

It may not, my dear Discipulus, be unnecessary to observe, that it would be to the advantage of the seller actually to cut the tree asunder in the line EF; because the butt, together with the limb, or branch, will form, what the shipwrights call a knee, which, therefore, is more valuable than timber of the common form. This I have already told you, but thought proper to repeat it, lest it should have slipped your memory.

Discip. I remember it very well, and was just going to make the same remark. But at the same time I should be glad to know why this sort of timber is so valuable.

Philo. Its value, is, in a great measure, owing to its scarcity, there being few woods that abound in this kind of timber; nor is the necessary care taken to propagate it.

Discip. Is it possible to propagate this sort of timber?

Philo. It is: at least, in some measure: we find, at present, upon heaths and other uncultivated places, a few straggling trees of oak. These seldom grow tall or regular; they were not properly defended from the injuries of cattle, and therefore browsed upon, and stunted while young, and thence become crooked, short-trunked, or pollard-trees. They are however, of great use to shipwrights, and fetch a good price at the markets.

As these trees, though in this respect far more valuable than tall strait timber, are taken very little care of, it is to be feared they will soon be all destroyed; it is, therefore, a very useful attempt to discover a regular method of producing others of a similar kind, and this may be easily done by following the same method which produced the former. They wholly owe their figure to the cattle's biting off their tops when young, and afterwards biting off the shoots from the first wound.

wound. If, therefore, a number of young trees set a part for the experiment, have their tops cut off at two, four, six, eight, ten, or twelve feet from the ground; and four years after, the shoots of these stunted trees be again cut in the same manner, they will grow up in all the irregular crooked figures that can be imagined; and, by pursuing this method, a supply of naturally crooked timber may be raised for the use of shipwrights with ease and certainty.

Discip. If I am not mistaken, I have heard some measurers mention the term *meeting* of a parcel of timber; I should be glad to know what they meant by that expression?

Philo. In order to understand this term, which is almost peculiar to the large yards of shipwrights, it must be remembered, that large timber is of much greater value than small. When, therefore, a quantity of timber is properly measured, they add the content of the several pieces together, and call that total the *meeting*. This being done, they consider what is the value of a foot of this lott of timber, one piece with another, and according to that value the whole quantity is sold.

Discip. Is this method just, both with regard to the buyer and seller?

Philo. No, certainly: a person not used to guess at the mean value of a parcel of timber, may be easily deceived by one of more experience.

Discip. Is there then no certain method by which a person may know the value of timber?

Philo. Doubtless there is: but a little experience is necessary, to know the worth of a single piece of timber. When this is known, let every tree be valued separately, and the several results added into one total. By this means, the danger of being deceived is avoided, and the work performed, with very little more trouble than by the other method.

S E C T. III.

Of the Mensuration of standing Timber.

Discip. Is it possible to measure timber while it is growing in the woods?

Philo. It may be done: but not without great trouble and fatigue, if accuracy be required. This, however, is rarely the case, unless the price of a tree that stands by itself, or one that is of remarkable value, be desired: an estimate only being thought sufficient for this purpose.

Discip. How am I to proceed in finding the solid content of a tree before it is cut down?

Philo. The only method of doing this is, to climb the tree, or get up it by means of a ladder; because it will be necessary to find the place where it measures only 24 inches in girt, after the proper allowance is made for the bark. Some authors, indeed, tell you, that it will be sufficient to measure the girt of the tree at the middle height to which it runs timber. This would, indeed, be sufficient, could the middle height be known: but as this cannot be discovered till the whole height be found, recourse must be had to the first method. When the place where the tree is no more than six inches the side of the square is found, the height or length of that part of the tree is easily known by the help of a line, having a plummet fastened to it, which you may easily let fall till it touch the ground, and then by tying a knot in the line, where it touches the upper girting place, you may measure the length between the lower end of the plummet and the knot in the line after you have descended from the tree, and that will be the length required. When you have thus obtained the quarter girt at the top, and the length of the tree, you must take the girt of the tree at the bottom, add the two girts together, (allowance being made for the bark) and take the half, which will be the mean girt, or that at the middle of the tree;

and consequently the solidity may be easily found by the methods already laid down for computing the content of tapering timber.

Discip. I can easily conceive this method; but the difficulty consists in putting it in practice. It will be both a tedious and a dangerous undertaking to measure the trees even in a small wood, and almost impracticable in a large one.

Phil. You are very right, Discipulus; and, therefore, this method is rarely practised. An estimate is generally thought sufficient; and this may be taken, with much less trouble.

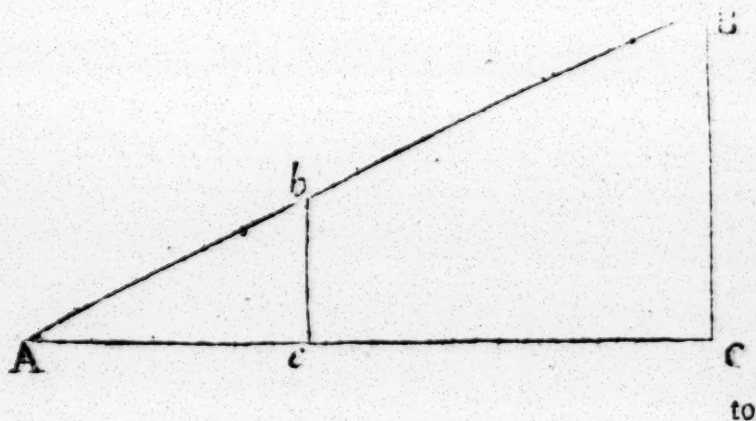
Discip. What are the methods most proper to be taken in forming estimates of standing timber?

Phil. Most of the methods commonly practised require the height of the tree, as far as it is timber, to be known; and, therefore, it will be necessary to shew you, in the first place, how this height may be found. In order to this, it will be necessary to explain a proposition or two relating to similar triangles.

Discip. What do you mean by similar triangles?

Phil. By similar triangles is meant, two, or more triangles, which have the same angles, and, consequently, their sides proportionable to one another.

Thus the two triangles in the annexed figure ABC , and Abc , are similar. For the angle at A , is common to both triangles, and the angle ABC is equal



to the angle Abc . Therefore, as $Ac : AC :: bc : BC$. The same proportion subsists between all the other parts of the two triangles.

If, therefore, we can, by any method, measure the lengths of the lines Ac , AC , and bc , we can find the length of the line BC , by calculation, without the trouble of actually measuring it.

Ex **EXAMPLE.**

Suppose the line Ac be 23.5 feet, AC 58, and bc 11.5, what will be the length of BC ?

SOLUTION.

$$23.5 : 58 :: 11.5 : 28.38$$

$$\begin{array}{r}
 58 \\
 \hline
 920 \\
 575 \\
 \hline
 23.5 \overline{) 667.0} 28.38 \\
 \underline{470} \\
 1970 \\
 \underline{1880} \\
 900 \\
 \underline{705} \\
 1950 \\
 \underline{1880} \\
 70
 \end{array}$$

Ans. 28.38 feet.

Discip. This is a very easy method, and I can easily perceive that BC may represent a tree, whose height is required; but how shall I find the lengths of Ac , AC , and bc ; and also know whether the two triangles are similar?

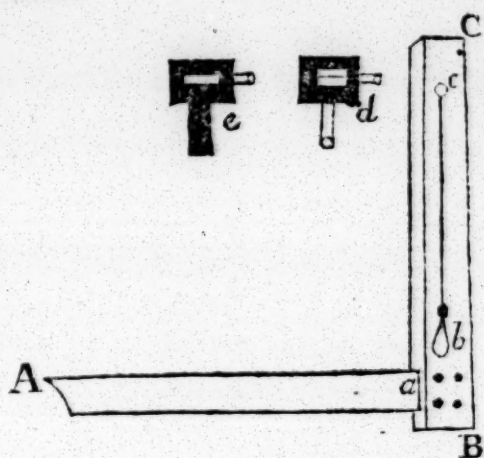
Philos. The difficulties contained in these questions I was just going to remove. In order to this, it may be necessary to observe, that AC represents the ground, which may easily be measured; BC the height of any object, as a tree, &c. and bc , a staff, of any known

length, erected in a perpendicular position on the ground. Now if we can, by any method, find when the point A, the top of the staff *b*, and the point B, are in a right line, the problem may be easily solved.

Disip. Thus far I understand the problem, and should be glad to know how this grand difficulty may be removed.

Philo. I will endeavour to give you satisfaction by describing a plain instrument, easily procured, and as easily applied to practice.

This is no other than a square made like that used by carpenters, but larger. The figure ABC, in the margin, will convey a sufficient idea of it. To this square belong two vanes or sights, like those on the common Gunter's

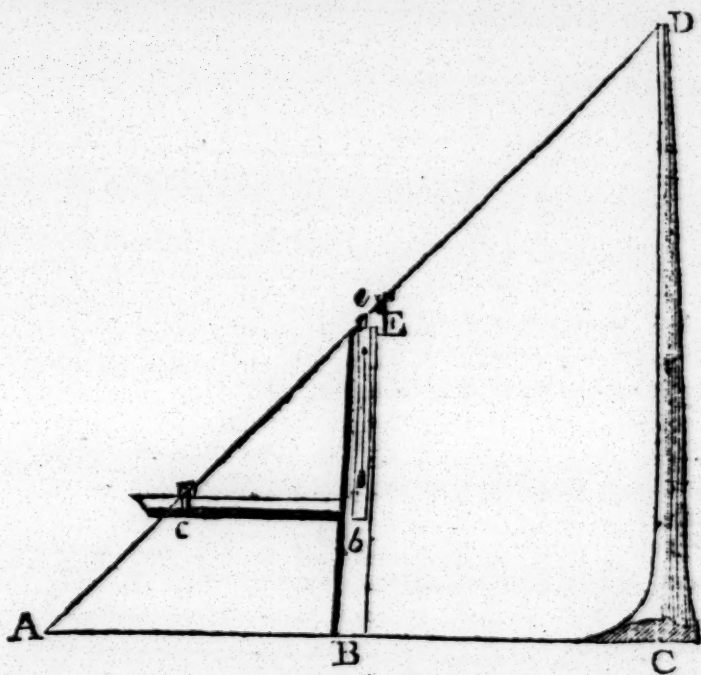


quadrant. The object vane *d*, is to be fixed in a hole made in the middle of the perpendicular limb of the square at C; and the sight vane *e* must slide, by means of a brass collet on the horizontal limb A *a*. To one of the sides of each of these vanes is fixed a small piece of brass, having a little hole at the extremity, sufficient for a silk thread to pass. The thread is fixed in the hole of the brass piece belonging to the object vane *d*, and inserted thro' the hole in the other. On the perpendicular limb a line and plummet *bc*, is hung from a pin at *c*, in order to keep it in a perpendicular situation. By the help of this simple instrument, the height of a tree, or any other object may be found in the following manner:

Suppose

Suppose it were required to find the height of the tree CD .

Erect your staff BE in some convenient place where the ground is level; fix your square bc upon your staff, in a perpendicular direction, by means of the plumb-line. This being done, and the staff firmly fixed in the ground, apply your eye to the sight-vane e , sliding it forward and backward, till you perceive the hair in the object vane at e cut the tree in point D . Fix the vane in that place, and extend the silk thread till it cuts the ground as at A . Measure, very accurately, the length of the line AB , AC , and the height of the staff, &c. Be . Then it will be as $AB : Be :: AC : CD$.



EXAMPLE.

Suppose the line AB the distance between the center of the staff and the place where the silk thread touched the ground) be 25.6-feet; the height of the staff,

P_4

Be

Be 26.5, and the line AC (the distance between the point where the silk thread touched the ground, and the foot of the tree) be 51 feet: what is the height of the tree CD?

OPERATION.

As 25.6 : 26.5 :: 51 : 52.79.

$$\begin{array}{r}
 51 \\
 \hline
 205 \\
 1325 \\
 \hline
 25.6 \overline{) 1351.5152.79} \\
 \underline{280} \\
 715 \\
 \underline{512} \\
 2030 \\
 \underline{1792} \\
 2380 \\
 \underline{2304} \\
 76
 \end{array}$$

Ans. 52.79 feet, or 52 feet 9 inches nearly.

From this operation you may observe, that if the height of the staff had been equal to the distance AB, the height of the object would have been equal to the distance AC. And as this point may be found by a few trials, you may, when you please, make use of this method.

Having thus found the height of the tree, you must ascend the tree to the height of about 26 feet, and there take the girt of it; whence the solid content of it is easily found, by the method and rules already laid down.

Discip. You have indeed entirely removed the difficulty; but I still think the measuring standing timber a very tedious and troublesome task. Is there no method of shortening the work, and yet forming an estimate near enough to answer the intended purpose?

Philo.

Philo The work may be greatly shortened in woods, where the trees are, in general, tall and strait, and taper equally from the bottom to the top. For if you find the height of one, you may guess at the rest, as they will be nearly proportional to the squares of their diameters.

The heights, and also the contents, of such tapering trees may be found by girding the tree at the bottom, adding to it 24 inches, the limit for timber, and taking the half of that sum for the true girt; for this will fall near the middle of the tree. When, therefore, this middle girt is found, and you ascend the tree, till you find the circumference equal to it; you may conclude, that you are near the middle of the tree. The figure in the margin will give you an adequate idea of this; where ACEFDB, represents the body of a tapering tree; the quarter of the girt at the butt AB is nine inches, and at the top EF, six inches. The sum of these two is 15 inches, whose half is 7.5; the quarter of the girt at CD. And this will always be the case in trees, that taper regularly from the bottom to the top.



If the heights are nearly the same, a very good estimate may be made by only taking the girt at the bottom, adding 24 inches to it, and taking the half of that sum as the true girt; for the square of one quarter of this girt multiplied by the length of a similar tree, found by the method already laid down, will give the content.

If the trees be of an unequal height but equally tapering, you may find the length near enough for practice by this proportion: as the girt at the bottom of one tree is to its length; so is the girt at the bottom of another to its length.

In this manner you may proceed to form an estimate of the whole content, and consequently the value of any number of trees that taper equally. When they do not taper equally, recourse must be had to actual mensuration by some of the methods already laid down; for the above proportion will not hold good in these cases.

CHAP. III.

Of measuring the tonnage of ships; or finding how much a ship will carry.

Discip. IS it not very difficult to find the tonnage of a ship? for if I am not greatly mistaken, her hold is a very irregular figure.

Philo. You are very right Discipulus; it would afford considerable employment for an able mathematician to solve the question accurately; but the practical method used by shipwrights is very easy and concise.

Discip. Is it customary for shipwrights to find the tonnage of ships?

Philo. It is: They generally undertake the building of them at so much per ton; and therefore it becomes absolutely necessary for them to find the tonnage.

Discip. What is the method they use.

Philo. Their method is this: they measure the length of the keel, the extreme breadth of the ship within-board, along the midship beam, and the depth of the hold from the plank joining the keelson upwards to the main-deck at the midship beam. These dimensions being taken, the tonnage is found by the following

RULE.

Multiply the length of the keel by the extreme breadth, and that product by the depth; the last product divided by 95 will give the burden in tons.

EXAMPLE

EXAMPLE I.

Suppose the length of a ship's keel be 60 feet; the extreme breadth 30 feet; and the depth 20 feet; what is her burthen or tonnage?

OPERATION.

$$\begin{array}{r}
 60 \\
 \underline{30} \\
 1800 \\
 \underline{20} \\
 95 \overline{) 3600} 37.894 \\
 \underline{285} \\
 750 \\
 \underline{665} \\
 850 \\
 \underline{760} \\
 900 \\
 \underline{855} \\
 450 \\
 \underline{380} \\
 70
 \end{array}$$

Anf. 37.894, or 38 tons nearly.

EXAMPLE II.

A captain contracts with a shipwright for building a ship of the following dimensions: the length of the keel is to be 112 feet 6 inches, the extreme breadth 60 feet 3 inches, and the depth 50 feet 9 inches; what will be the value of the ship at 5*l*. 10*s*. per ton?

THE WHOLE ART

OPERATION.

$$\begin{array}{r}
 112.5 \\
 \underline{60.25} \\
 5625 \\
 2250 \\
 \underline{67500} \\
 6778.125 \\
 \underline{50.75} \\
 33890625 \\
 47446875 \\
 \underline{33890625}
 \end{array}$$

$$951343989.8437513620.94972 \text{ Tons.}$$

$$\underline{285 \dots \dots \dots}$$

$$\cdot 589$$

$$\underline{570}$$

$$\cdot 198$$

$$\underline{190}$$

$$\cdot \cdot 858$$

$$\underline{855}$$

$$\cdot 434$$

$$\underline{380}$$

$$\cdot 543$$

$$\underline{475}$$

$$\cdot 687$$

$$\underline{665}$$

$$\cdot 225$$

$$\underline{\cdot 190}$$

$$\underline{\cdot 35}$$

$$362094572$$

$$\underline{5.5} \text{ Price per ton.}$$

$$1810472860$$

$$\underline{1810472860}$$

$$19915.201460$$

Ans. The burden is 3620.94572 tons, and the amount
 19915*l.* 4*s.* 0*d.*

Discip.

Discip. Is this method true, and the result accurate and just?

Philo. It is not: the solid content in feet, resulting from multiplying the length, breadth, and depth together, is much too large: but being divided by a number also much too large, the quotient, which gives the tonnage of the ship, is tolerably near the truth.

Discip. That is, if I understand you right, one error is corrected by another. Would it not then be better to form another rule from undoubted principles, that would give the same result without having recourse to error?

Philo. Certainly it would: But this is impossible; there can no general rule be formed, that may be applied to ships in general. Nor can the hold of a ship be measured but by piece-meal, and even the solid content of several of these parts can only be found by approximation.

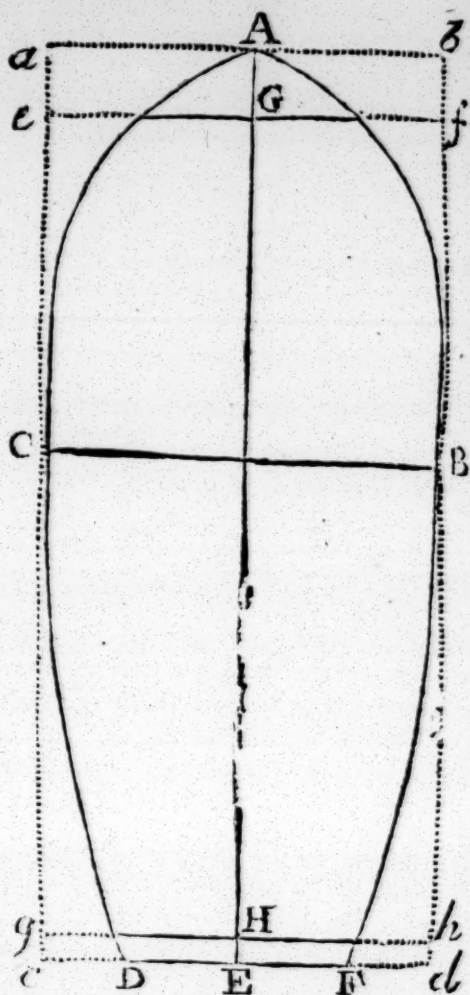
Discip. Why can no general rule be formed for finding the tonnage of all kinds of ships?

Philo. Because the curves, which form the bottoms of ships, are, at once, very irregular, and very different. Consequently, two ships may be of the same dimensions, with regard to length, breadth, and depth, and yet one of them carry many more tons than the other.

Discip. I am greatly obliged to you, Sir, for this explanation: but you said, that the result of the length, breadth, and depth, multiplied into one another, gave the solid content in feet, much too large; and that, by dividing the last product by a number also much too large, the error was, in some measure, corrected. I should be glad if you would be a little more explicit with regard to these assertions; for though you have convinced me that it is impossible to form any general rule that will give the true solid content of ships' holds in general, yet I should be glad to know whence these errors arise, and why they, in some measure, correct each other.

Philo. Your curiosity is highly laudable, and I will endeavour to give you all the satisfaction in my power.

In order to this, let the curve superficies $ABCDE$ F , represent the deck of a ship; then will CB be the extreme breadth, AE the length, and GH the length of the keel. But if GH be multiplied by CB , you will have the area of the parallelogram $efgh$, instead of the area of the curve superficies. The difference will be still greater with regard to depth, because the bottom of a ship is entirely formed of curve lines; but the solidity found by the last multiplication is that of a parallelopiped, in which the entire hold of the ship, supposing it



every where equal in breadth with the deck, is contained; consequently the solidity resulting from the length, breadth, and depth into one another is much greater than that of the hold of a ship.

But this solidity is divided by 95, which greatly reduces the quantity of tonnage. You will be easily convinced of this when you are informed that it has been found by a variety of experiments, that a cubic foot of rain water weighs 62.5 pounds. If therefore

2240 ($= 112 \times 20$) the number of pounds in a ton, be divided by 62.5, we shall have 35.8 for a divisor, by which the content found above, supposing it right, should have been divided. But 95, the common divisor, is considerably more than double 35.8. Consequently, more than half the content is allowed for the difference between the form of a ship's hold, and that of a parallelopiped.

Discip. But will this divisor balance the former error; and can there be any reason given why the number 95 was pitched upon, rather than any other?

Philo. The divisor is not sufficient to balance the error; nor can there be any reason given why the number 95 was chosen; it was, probably, introduced at a time when the mathematical sciences were little cultivated, and the principles of Hydrostatics hardly known; and will, in all probability, be continued till some more exact number be found out and established by authority. But though I can give you no satisfaction with regard to the latter part of your question, I will point out a method by which the weight any ship, bark, or boat can conveniently carry, may be estimated very near the truth.

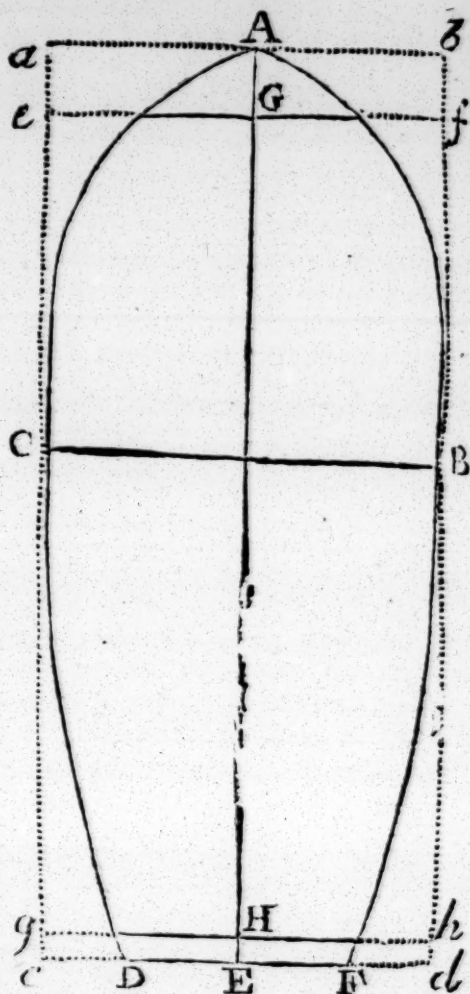
Discip. I shall esteem it a very particular favour, as the subject is curious, and may, perhaps, be useful.

Philo. It is, undoubtedly, a curious enquiry, and, therefore, merits attention, even though it should never be practised.

It is evident, by the laws of Hydrostatics, that the weight of a floating body is equal to as much water as is displaced by it; consequently, an entire ship with all her loading, &c. is equal to as much water as will form a solid in all respects similar to that part of the ship, which is below its surface; or to that bulk of water that is displaced by the ship.

Hence it is evident, that the difference between the bulks of water, displaced by the ship when light and when loaded, will be the true tonnage of the ship. In order to discover what this bulk of water is, let the line, formed by the surface of the water when the ship is light, be marked on the head and stern: and
also,

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Hence it is evident, that the difference between the bulks of water, displaced by the ship when light and when loaded, will be the true tonnage of the ship. In order to discover what this bulk of water is, let the line, formed by the surface of the water when the ship is light, be marked on the head and stern: and
also,

also, another when she is sufficiently laden. This being done, let a parallelopiped be formed proportional in length to that of the ship's keel; in breadth, to that of the extreme breadth of the ship; and in depth, to that of the deep load mark. Find exactly the solid content of this parallelopiped, and also weigh it accurately in a balance. Fashion the parallelopiped as nearly as possible into the form of the ship's bottom from the keel to the deep load-mark, and weigh it again very carefully. When this is performed, the following proportion will solve one part of the question. As the first weight of the parallelopiped is to its content; so is the weight of it after being fashioned to its content. This result, divided by 35.8, the number of solid feet contained in one ton of water, will give the tonnage of the ship, including the weight of her hull, &c.

EXAMPLE.

Suppose the parallelopiped, measured on a scale proportioned to the experiment, be 168.5 feet in length, 48 feet in breadth, 20 feet in depth, and that it weighs 40 pounds 13 ounces: but when formed into a model of the ship, it weighs only 30 pounds 3 ounces: what is her burden?

OPERATION.

168.5 Length.
48 Breadth.

13480
6740
8088.0

20 Depth.

161760.0 Content.

lb	oz		lb	oz
As 40	13	:	161760	:
16		:	483	:
<u>653</u>			<u>16</u>	
			<u>483</u>	

1294080
647040
653 | 78130080 | 119647.9

653.....

1283

653

• 6300

5877

• 4230

3918

• 3128

2612

• 5060

4471

• 5890

5877

• 13

35.8|119647.9|3342.12

$$\begin{array}{r}
 1074 \cdot \cdot \cdot \\
 \hline
 \cdot 1224 \\
 1074 \\
 \hline
 \cdot 1507 \\
 1432 \\
 \hline
 \cdot \cdot 759 \\
 716 \\
 \hline
 \cdot 430 \\
 358 \\
 \hline
 \cdot 720 \\
 719 \\
 \hline
 004 \\
 \hline
 \end{array}$$

Anf. 3342.12 tons; or 3342 tons, 2 hundred and 45 pounds nearly.

Let us now compute the weight of the hull of a ship, with her rigging, sails, &c. that is the weight that sinks her to her light draught of water.

In order to this, let another parallelopiped be procured, whose length is proportional to that of the ship at the light draught of water, its breadth proportional to her breadth there, and its depth proportional to the ship's depth from that line to her keelson. Weigh the parallelopiped very accurately, and form it into the model of the ship's bottom. This being done, weigh the piece again with the same degree of accuracy; and the above proportion will give the content in feet, and being divided by 35.8, the tons required.

EXAMPLE.

Suppose the dimensions of the above ship be these: length 163.75; breadth 47, and depth 12.75 feet. Let also the weight, when rough, be 27 pounds, 6 ounces; and when fashioned, 18 pounds, 8 ounces; and let the tonnage be required.

OPERATION.

163.75 Length.
47 Breadth.

114625
65500

7696.25
12.75 Depth.

3848125
 5387375
 1539250
769625
 98127.1875 Content.

lb oz : 98127.1875 : : lb oz
 As 27—6 : 296 : : 18—8 : 66314.6
16 16

168 5887631250
27 8831446875
438 1962543750

116
18
296

438 | 29045647.50001
 2628.....

• 2765
2628

• 1376
1314

624
438

1867
1752

• 1155
876

• 2790
2628

• 162

THE WHOLE ART

35.8|66314.26|1852.35

$$\begin{array}{r}
 358 \dots \\
 \hline
 3051 \\
 2864 \\
 \hline
 \cdot 1874 \\
 1790 \\
 \hline
 \cdot \cdot 852 \\
 716 \\
 \hline
 1266 \\
 1074 \\
 \hline
 \cdot 1920 \\
 1790 \\
 \hline
 130
 \end{array}$$

Ans. 1852.35 tons.

From the tons necessary to sink the ship to } 3342.12
 her deep load mark.

Take the weight of the hull, &c. } 1852.35

Remains of the tons of loading the ship } 1489.77
 will carry.

Discip. This seems a far more accurate method than the former, for finding the burthen of a ship: but is it not difficult to shape the parallelopiped into the model of the ship?

Philo. It certainly is: and accordingly is hardly ever put in practice. I only mentioned it as a curious experiment, which might be used if the practitioner thought proper.

Discip. But is this at all applicable to shipwrights? can they measure their vessels by this method?

Philo. Certainly: provided the parallelopiped could be truly formed: But as this would require the utmost accuracy, it is never attempted.

C H A P. IV.

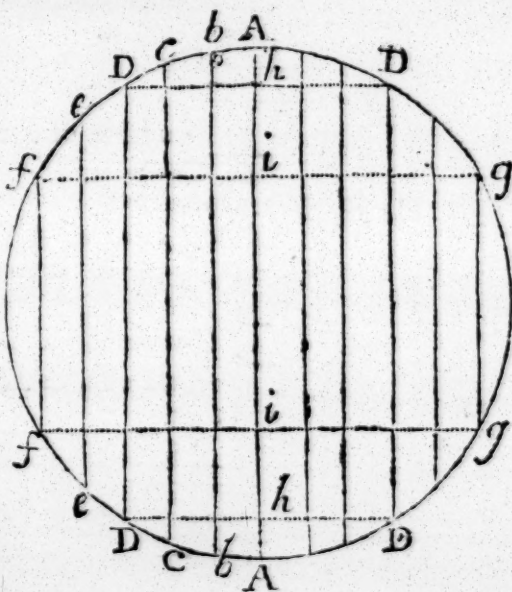
Of measuring Sawyer's Work.

Discip. IS there any thing particular in measuring Sawyer's Work?

Philo. There is, if you follow what is generally called Sawyers Measure: They take the diameter of the piece that is to be cut into boards, and multiply that diameter into the length, which gives the superficial content; this result they multiply by the number of cuts, and consider the product as the true content of their work, which they are generally paid for by the hundred feet.

Discip. This must certainly be unjust; because neither of the cuts can be so broad, as that which goes through the center of the tree.

Philo. You are extremely right; and in order to demonstrate the error resulting from this method of measuring, I have drawn the figure placed in the margin, where AA is the cut whereby the whole is measured according to the method used by sawyers; and for which they are paid at London 3s. per hundred feet superficial measure, if the timber be oak.



If

If AA be 1 foot, then will *bb*, be 11.75 inches, *cc* 11.5, DD 10.75, *ee* 9, and *ff* 7 inches. From these dimensions let us see what will be the difference between the superficial content of the several cuts when measured truly, and when the result is found according to the practical method.

EXAMPLE.

Suppose a piece of timber 15 feet long, and 1 foot in diameter, were to be cut into 10 boards and 2 slabs, all of an equal thickness, and the lengths of the cuts as mentioned above; what will be the superficial content of the whole, measured by the sawyers method, and what will be lost by substituting that rule instead of the true.

OPERATION.

I. By Sawyers measure.

15 Length.
1 Cut.

15
11 Number of cuts.
165 Content.

II. By the true method.

Middle cut, or AA.

15
1

15 Content in feet.

Second cut, or *bb*.

11.75

15

5875

1175

176.25

2 Number of equal cuts.

352.50 Content in inches.

Third

Third cut, or *cc*.

$$\begin{array}{r} 11.5 \\ 15 \\ \hline 575 \\ 115 \\ \hline \end{array}$$

17.25

2 Number of equal cuts.

$$\begin{array}{r} 345.0 \\ \hline \end{array} \text{Content.}$$

Fourth cut, or *DD*.

$$\begin{array}{r} 10.75 \\ 15 \\ \hline 5375 \\ 1075 \\ \hline \end{array}$$

161.25

2 Number of equal cuts.

$$\begin{array}{r} 322.50 \\ \hline \end{array} \text{Content.}$$

Fifth cut, or *ee*.

$$\begin{array}{r} 15 \\ 9 \\ \hline \end{array}$$

135

2 Number of equal cuts.

$$\begin{array}{r} 270 \\ \hline \end{array} \text{Content.}$$

Sixth cut, or *ff*.

$$\begin{array}{r} 15 \\ 7 \\ \hline \end{array}$$

105

2 Number of equal cuts:

$$\begin{array}{r} 210 \\ \hline \end{array} \text{Content.}$$

Content of the 2d cuts	—	352.5
3d ditto	—	345
4th ditto	—	322.5
5th ditto	—	270
6th ditto	—	210
		12 1500 0 125
		12 . . .
		30
		24
		60
		60

Content of 2, 3, 4, 5, 6 cuts	125
of the middle cut	<u>15</u>
	140
By Sawyers measure	165
By the true method	<u>140</u>
Difference	<u>25</u>

Cont. of the whole

Ans. The content by sawyers measure is 165 feet, and the loss by using that method is 25 feet.

Discip. Might not a multiplicator be found, which would give the true content by a single operation?

Philos. Undoubtedly: and you have nothing more to do than to add the depth of the several cuts together, and multiply that sum by the length, for the product will give the content required.

	I.
AA ———	12
bb × 2 ———	23.5
cc × 2 ———	23.
DD × 2 ———	21.5
ee × 2 ———	18
ff × 2 ———	14
Sum	112
Length	15
	560
	112
	12 1680 140.
	12
	48
	48
	00

Ans. 140 feet, the same as before.

I intended to have solved the question in this manner, tho' you had not requested it ; for you must have observed, that in the whole course of my instructions I have solved all the questions in the plainest manner, in order to avoid a fault too common in writers on these subjects, who, by endeavouring to deliver their solutions in a neat and concise manner, give the learner an infinite deal of trouble. I have, therefore, given my solutions in the plainest, though often not in the concisest manner. I am willing to sacrifice brevity to perspicuity ; and shall think myself happy if my pupils thoroughly understand my precepts ; contractions and concise methods of solving the same question will readily occur, when a person thoroughly understands the reasons on which each rule is founded.

Discip. I am abundantly convinced of the truth of your observation ; but I intended to ask, whether any one of the cuts, multiplied by the length, would give the true content ; or whether a mean multiplicator might be found, which would solve the question.

Philo. It is not impossible, but very tedious to find a common multiplicator: an inspection of the figure will sufficiently inform you, that the depths of the cuts do not decrease in arithmetical progression; for the difference between the depths of the cuts DD , and AA , is much less than that between ff and DD ; notwithstanding there are two intermediate cuts between the former, and one only between the latter. That is, the length of the line hi , intercepted between the dotted lines DhD and fig , is much greater than Ab . In fact, the cuts are chords of a circle, and separate parts of an unequal number of degrees. For the limb of the circle fe , intercepted between the cuts or chords ff , ee , is much greater than bA , intercepted between bb , and AA ; notwithstanding they are at equal distances, with regard to the diameter.

But though an accurate multiplicator cannot be readily found; yet if the mean cut, or that at equal distances between the greatest and least cut, be taken, and the length be multiplied by it, and that product multiplied by the number of cuts, the result will be much nearer the truth than that found by *Sawyers measure*; and the content found by this mean cut is what is called *market measure*. Let us see what will be the content of the above timber by this method.

$$\begin{array}{r}
 10.75 \text{ DD.} \\
 15 \text{ Length.} \\
 \hline
 5375 \\
 1075 \\
 \hline
 161.25 \\
 11 \text{ Number of cuts.} \\
 \hline
 1211773.75 | 147.81 \text{ Content.} \\
 12 \dots \dots \\
 \hline
 .57 \\
 48 \\
 \hline
 .93 \\
 84 \\
 \hline
 .97 \\
 96 \\
 \hline
 .15 \\
 12 \\
 \hline
 .3
 \end{array}$$

The content found by this method is 7.81, or almost 8 feet too much; but 17.19 feet less than that found by Sawyers measure, and consequently nearer the truth by that quantity.

Discip. Is it not surprizing, that so erroneous a method should prevail in London, where there are such numbers of persons both able and willing to correct it; especially as another method is both known and practised elsewhere?

Philo. It is certainly strange; but ignorance has introduced, and custom established many errors, and even absurdities, which, it is to be feared, will not soon be eradicated. I will give you another instance of this kind.

The sawyers at London have, by custom, as much per load, one piece with another, for breaking down, or cutting a piece of timber, into large scantlings, as they have for breaking down, or cutting it into small scantlings. That is, they will cut a piece of timber a foot square, of any length, into two scantlings of 12 inches by 6; or into 4 scantlings of 6

Q 2

inches

inches by 6; or into 12 scantlings of 4 inches by 3, all at the same price.

Discip. This is really astonishing! can they give any reason for so absurd a practice?

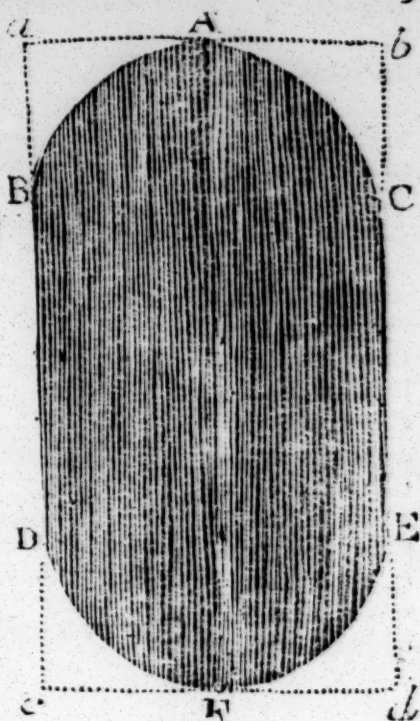
Philo. They have, what they call a reason, for this preposterous method; it is this; They say, that where the timber proves fine it is seldom, perhaps never, the interest of the owner, to have it cut into small scantlings. This reason is sufficient to continue the practice, though they constantly see instances to the contrary, and are often obliged to work for whole days at half price.

C H A P. V.

Of finding the content of goods for loading of ships, and which pay freight by the solid or cubic foot.

IN order to estimate the freight of goods, it will be necessary to take the dimensions of the several cases, bales, trusses, casks, &c. in a very accurate manner. The ship's measurer is generally very careful, for otherwise he will lose a considerable part of the freight. In order to this, the particulars should be attentively examined, and if of any irregular figure, the dimensions must be taken, in the largest parts, and paid for according to that result.

The figure in the margin will more clearly explain this remark. $ABDFEC$ represents the plan of a case filled with goods, for which a certain price for every solid foot is to be paid as freight. But in measuring this, no regard is to be paid to the two circular ends; the case must be considered as a parallelopiped, and consequently its base as a parallelogram that is, the base as equal to $abcd$; and the area found by multiplying $ac = AF$ into $cd = ab$; the same must be observed with regard to its height.

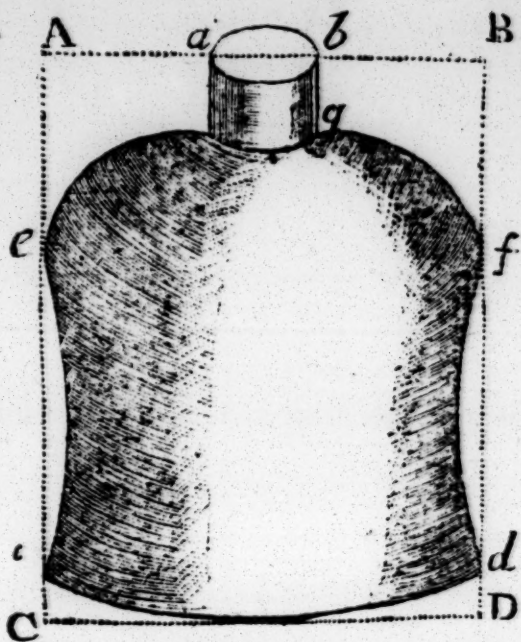


Discip. But is not this method of proceeding very unjust? why should any more than the real content be paid for?

Philos. Because no goods of any consequence can be stowed in the vacant places aAB , AbC , DcF , and $E dF$. They are lost to the captain of the ship, unless paid for by the owner of the goods.

Discip. Are all goods measured and paid for in this manner?

Philos. Yes; even large stills are not excepted, tho' the vacancies are much greater than in bales, casks, &c. The figure in the margin will be sufficient to shew you the method used in these cases: *ab* *cdef*, represents the body of a still, supposed to be conigned to a person abroad, and to be measured,

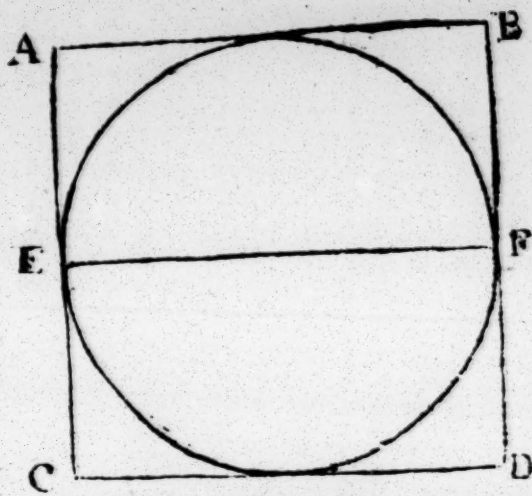


that the freight may be ascertained. In order to this, the ship's measurer takes the greatest diameter, as at *ef = cd*. He even measures round the nails, if there are any in the place where the greatest diameter is found. This diameter is considered as the side of a square, and multiplied by itself, which gives the superficial area of the base, that the bottom of the still is supposed to take up in the ship's hold; and this area multiplied by the greatest height as $AC = BD$, gives what is called the solid content. By this means all the vacancies, as *Aae*, *bBf*, &c. besides those at the bottom, are taken in, and the whole paid for, as if it were a bale of cloth, &c. in the form of a parallelopiped, of the above

dimensions.

Dimensions.

The figure in the margin represents a horizontal Section or plan of the bottom of the still, where EF , the diameter of the circle and equal to CD , is considered as the side of the square $ABCD$. Let



us see what will be the content of this still, measured according to the above method.

EXAMPLE.

Suppose CD 6.8 feet, BD (in the first figure) 8.5, and let it be required to find the content.

OPERATION.

$$\begin{array}{r}
 6.8 \\
 6.8 \\
 \hline
 544 \\
 408 \\
 \hline
 46.24 \\
 8.5 \\
 \hline
 23120 \\
 36992 \\
 \hline
 393.040 \text{ Content.}
 \end{array}$$

Discip. I imagine, Sir, you have omitted a very material circumstance: The body of the still is not solid; should not that particular be considered in the measurement?

Phil. It certainly ought to be considered, were the true solidity of the still required; but the case is very different here; the question is, how much room it will take up in a ship's hold. And the ship's measurers are so very indifferent with regard to its being solid or not, that the person who sends the still is at liberty to fill it with corn, or any other goods he pleases, without any additional expence, with regard to freight.

C H A P. VI.

Of measuring Cord-wood.

Discip. **W**HAT do you mean by cord-wood?

Phil. By cord-wood is meant pieces of wood of different sizes, but generally pretty large, cut of the same length, and piled up very carefully, in order to be measured, and afterwards sold by the cord.

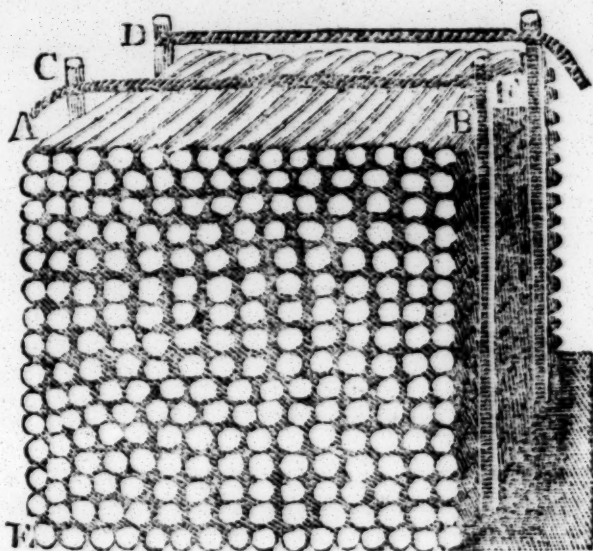
Discip. What do you mean by a cord of wood?

Phil. A cord of wood is a pile of the above pieces: but the dimensions of this pile are different, in different parts of England. What is called a statute cord is a pile eight feet long, four feet broad, and four feet high; and, consequently, its content is 128 feet; for $8 \times 4 \times 4 = 128$. This sort of cord is used in most of the northern counties of England; but in Sussex, and most of the southern counties, a pile of wood 3 feet high, 3 feet wide, and 14 feet long, is called a cord. The content of this is 2 feet less than that of the other, for $14 \times 3 \times 3 = 126$.

Discip. Are all the piles of these dimensions: and what methods are taken to make them so?

Phil. They are not all of these dimensions; for if they were, there would be no occasion to measure them. The labourers make their piles larger or smaller, according as the quantity of wood is greater or lesser. All they take care of is to cut the pieces all of the same length, or nearly so, and to pile them
up

up in a very regular manner. Accordingly, they pile the pieces up in so close a manner, that the vacuities between them are very small; and to support the pile in a perpendicular situation, they drive four stakes into the ground, and fasten the tops of them with cords. The figure in the margin will sufficiently explain this; where A E is a pile of wood supported in a perpendicular situation by the four stakes C D B F, fastened together by the cords C B, D F.



All the wood that has been felled for that purpose being thus formed into piles, the next thing is to have it measured for sale. In order to which the dimensions of each pile are taken very accurately, and the content found by the following

R U L E.

Multiply the length by the breadth, and that product by the height : the result divided by 128 (or the content of a cord) will give the content required.

Suppose a stack of wood, piled up in the above manner, be 25 feet long, 18.66 wide, and 5.5 feet high;

THE WHOLE ART

how many cords of wood does it contain, reckoning
128 feet to the cord?

OPERATION.

18.66 Breadth.
25 Length.

9330

3753

466.50

5.5 Height.

23325

23325

128 | 2565.75 | 20.04

256...

... 575

512

.63

Ans. 20.04 cords.

EXAMPLE II.

Suppose a stack of wood be 30 feet 6 inches long,
20 feet 3 inches broad, and 4 feet 9 inches high; how
many cords does it contain; reckoning 126 feet to the
cord?

OPERA-

OPERATION.

$$\begin{array}{r}
 20.25 \text{ Breadth.} \\
 \underline{30.5 \text{ Length.}} \\
 10125 \\
 60750 \\
 \underline{617.625} \\
 4.75 \text{ Height.} \\
 \underline{3088125} \\
 4323375 \\
 \underline{2470500} \\
 12612933.71875 | 23.2834 \\
 \underline{252} \\
 413 \\
 \underline{378} \\
 357 \\
 \underline{252} \\
 1051 \\
 \underline{1008} \\
 438 \\
 \underline{378} \\
 597 \\
 \underline{504} \\
 35
 \end{array}$$

Anf. 23.2834 cords, or something more than 23 cords and one quarter.

CHAP. VII.

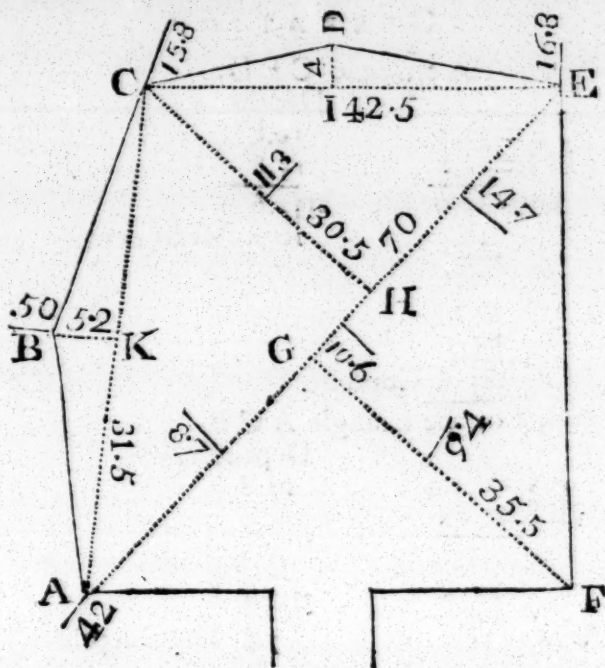
Of measuring Marle pits.

Discip. **I**S it customary to measure marle pits? *Philo.* In many counties of England, as Lancashire, &c. where marle is found in great plenty, it is: and because I would not have you ignorant of any part of mensuration, I imagined it might not be improper to explain the method of proceeding in such cases.

Marle is known to be an excellent manure, and accordingly is used with great success by the Farmers in every county where it is found. In Lancashire, where this practice is general, they pay about twenty shillings a rod for digging and spreading the marle on the surface of the land: but the rod in that county is 8 yards, or 24 feet, which is 7.5 feet greater than the statute rod. Consequently, 64 cubic yards, or 1728 cubic feet is a rod of marle in that county. When the work is finished, the pit is measured, and the marlers paid for the quantity taken out of it. The mensuration is performed in the following manner.

They first measure the surface of the pit, and plot it, that is, draw the form of it upon paper, and divide the plot into trapeziums and triangles, as in surveying

land. This being done, they measure the depths in different places of the different places of the Pit, and set them down upon the plot, as you see is done in the annexed figure. This being finished the content is found by the following



R U L E.

Find the superficial area of the several triangles, trapeziums, &c. and multiply the area of each by its mean depth; and the sum of all these, divided by the proper divisor, will give the content of the whole pit.

E X A M P L E.

Suppose the content of the marle pit, represented in the above figure, and of the dimensions set down on the respective lines, &c. was to be measured; how many solid rods will it contain; and what will the expence

pence amount to at nineteen shillings and six pence the rod?

N. B. The numbers that express the depths, have lines drawn either over or under the figures.

OPERATION.

First, For the triangle CED.

CE=42.5	Depths.
$\frac{1}{2}$ DI= 2	16.8
Area 85.0	<u>15.8</u>
16.3	2 32.6
<u>255</u>	16.3 Mean depth.
510	
<u>85</u>	
1385.5	

Content.

Second, For the triangle ACB

AC=51.5	Depths.
$\frac{1}{2}$ BR= 2.6	15.8
3090	5.0
<u>1030</u>	<u>2.1</u>
Area 133.90	3 22.9
7.6	7.6 Mean depth.
<u>8034</u>	
9373	
<u>1017.64</u>	

Content.

Third

OF MEASURING.

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Third, For the trapezium ACEF.

Depths.

AE=70	CH=30.5	16.8
<u>33</u>	CF=35.5	<u>15.8</u>
210	Sum 66.0	14.7
<u>210</u>	$\frac{1}{2}$ Sum 33.	11.3
Area 2310		10.6
<u>10.3</u>		9.4
6930		8.7
<u>23100</u>		3.4
23793.0	Content.	<u>2.1</u>
		9192.8
		10.3 Mean.

Content of ACEF=23793

ACB= 1017.64

CED= 1385.5

1728|26196.14|15.159

1728

8916

8640

2761

1728

10334

8640

16940

15552

1388

As 1 Rod : 195. 6d. = 975l. :: 15.159 :

975

75795

106114

136431

14.780025

20

15.60

12

7.2

Ans.

Ans. The content of the Pit is 15.159 rods; and the expence 14*l.* 15*s.* 7*d.*

Discip. Does not the surface of the marle lie at some distance below the surface of the ground; and if so, is the depth to be taken from the surface of the ground, or from the surface of the marle?

Philo. There are few instances where the upper layer or stratum of the marle is not below the surface of the ground; it is often found at about two feet below the surface: but whatever the depth of the earth may be, it is never considered in measuring the pit; the depths are taken from the surface of the marle to the bottom of the pit, without the least regard to the earth, or mould above it.

CHAP. VIII.

Of measuring Hay Ricks.

DISCIP. Is it possible to find the quantity of hay contained in a rick by mensuration?

Philo. It is, perhaps, impossible to find the true content of a stack of hay, because one rick of hay may contain more loads than another of the same dimensions, according to the goodness of the hay, and the care taken in laying it up. But though accuracy must not be expected, yet a very good estimate may be made, and the value of a stack of hay in ricks computed very near the truth.

Discip. What methods must be taken to compute the contents of hay-ricks?

Philo. The dimensions must be carefully taken in feet and inches, &c. and the content of the solid computed by some of the foregoing rules already laid down in this treatise; for this content divided by the number of solid feet in a load, will give the number of loads contained in the rick.

Discip. Is it known how many solid feet make a load of hay?

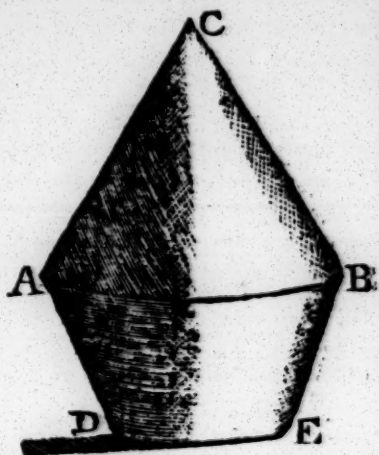
Philo.

Philos. The precise number cannot always be known; and this renders it impossible to compute the number of loads contained in the rick. For could this number be known, the content of a hay-stack in loads, might be computed with sufficient accuracy. A load of hay contains 36 trusses, each truss weighing 56 pounds, or half a hundred weight; so that 18 hundred weight is a load of hay. But the contents of the trusses are various; some of them containing near 14 solid feet, and others little more than 7, though both trusses are of the same weight. However, if 10 or 11 solid feet be considered as a mean, the computation will not be far from the truth. Upon this foundation, a load of hay will contain about 360 or 400 solid feet.

When the number of solid feet in a truss, or a load are determined, the content of the hay-stacks must be found; and in order to this, the form of the solid must be considered. There will be no great difficulty in this, as all hay-stacks stand either on a rectangular or a circular base; and are composed of two tapering solids, whose content will be easily found by the rules already laid down. This will sufficiently appear, if the two sorts of ricks be considered separately.

1. *Of the circular Hay-Rick.*

The figure in the margin represents a circular hay-rick; and it is sufficiently evident from the figure, that the whole stack is composed of two tapering solids ABC , and $ABDE$. The upper is a cone, standing upon its proper base; and the latter, a frustum of a cone, standing upon its lesser base.



But the solid content of the cone ABC , is easily found by the rule laid down page 145: and that of the frustum $ABDE$, by either of the rules mentioned page 157. Hence the content of the whole may be easily found. And this content divided by 360, or 400, according as the hay has been well or ill made, will give the number of loads required..

EXAMPLE I.

Let it be required to find how many loads of hay are contained in the above rick, supposing the diameter of the thickest part of the rick AB , 30 feet 6 inches, the perpendicular height of the cone ABC 20 Feet 3 inches; the lesser diameter DE of the frustum $ABDE$. 24 feet 9. inches, and its perpendicular height 8 feet.

1. For

OF MEASURING.

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1: For the upper Solid.

Operation. See Page 145, 146.

$$\begin{array}{r}
 30.5 \\
 \underline{30.5} \\
 1525 \\
 \underline{9150} \\
 930.25 \text{ Square of A B.} \\
 \underline{7854} \\
 372100 \\
 465125 \\
 744200 \\
 \underline{651175} \\
 730.618350 \text{ Area of the Base.} \\
 \underline{6.75} \text{ One 3d of the Height.} \\
 365309175 \\
 511432845 \\
 \underline{438371010} \\
 4100)4931.6738625(12.3291846 \text{ Loads.} \\
 \underline{4} \\
 .9 \\
 \underline{8} \\
 13 \\
 \underline{12} \\
 11 \\
 \underline{8} \\
 .36 \\
 \underline{36} \\
 .7 \\
 \underline{4} \\
 33 \\
 \underline{32} \\
 .18 \\
 \underline{16} \\
 .26 \\
 \underline{24} \\
 .225
 \end{array}$$

2 For

2. For the lower Solid.

Operation. See Rule 2, Page 157.

$$\begin{array}{r}
 24.75 \\
 30.5 \\
 \hline
 12375 \\
 74250 \\
 \hline
 754.875 \\
 10.594 \text{ Add.} \\
 \hline
 765.469 \\
 .7854 \\
 \hline
 3061876 \\
 3827345 \\
 6123752 \\
 5358283 \\
 \hline
 601.1993525 \\
 8 \\
 \hline
 4.00 \overline{) 4809.59482108} \\
 12.0239870 \\
 \hline
 12.3291846 \text{ Cone.} \\
 12.0239870 \text{ Frustum.} \\
 \hline
 24.3531716 \text{ Loads.}
 \end{array}$$

Anf. 24 Loads, 1 Quarter, and a Half, nearly.

EXAMPLE II.

How many Loads of Hay are contained in a circular Hay-rick, whose largest Diameter is 60 Feet, its lesser 40; the Perpendicular Height of the Upper Solid 50 Feet, and that of the lower 20 Feet: and what will the Whole amount to at 2/. 5s. per Load?

OPERA-

OPERATION.

$$\begin{array}{r}
 60 \\
 \underline{60} \\
 3600 \\
 \underline{.7854} \\
 14400 \\
 18000 \\
 28800 \\
 \underline{25200} \\
 2827.4400 \text{ Area of the Base.} \\
 16.66 \text{ One third Height.} \\
 \underline{1696464} \\
 1696464 \\
 1696464 \\
 \underline{282744} \\
 47105.1504 \text{ Content upper Solid.}
 \end{array}$$

$$\begin{array}{r}
 60 \\
 \underline{40} \\
 2400 \\
 \underline{133.33 \text{ Add.}} \\
 2533.33 \\
 \underline{.7854} \\
 1013332 \\
 1266665 \\
 2026664 \\
 \underline{1773331} \\
 1989.677382 \\
 \underline{20} \\
 39793.547640 \text{ Content lower Solid.} \\
 47105.1504 \text{ Upper Solid.} \\
 \underline{4|00)86898.698(c4} \\
 217.24674 \text{ Loads.}
 \end{array}$$

As

As 1 Load 2l. 5s. (≈ 2.25) :: 217.24674 :

$$\begin{array}{r}
 2.25 \\
 \hline
 108623370 \\
 43449348 \\
 43449348 \\
 \hline
 £488.8\text{c}, 1650 \\
 20 \\
 \hline
 s\ 16.103300 \\
 12 \\
 \hline
 d\ 1.2396
 \end{array}$$

Ans. The Content of the Rick is 217.24674 or something more than 217 Loads, and the value 488l. 16s. 1d.

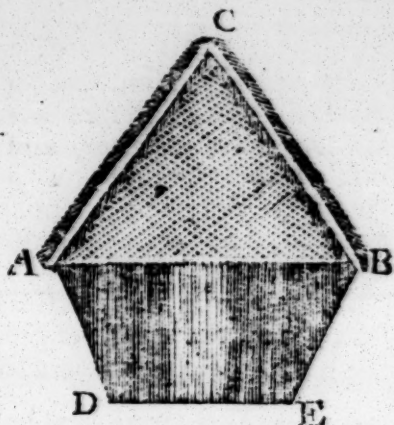
Discip. Is this the Method generally practised in measuring Hay-stacks of a circular Figure?

Philo. I do not remember to have ever seen any Method practised: experience is the general Guide, and the Contents of Hay-stacks are commonly guessed at by those who have been long conversant in Husbandry.

Discip. This must surely be a very precarious Method; and I should imagine that a Stack of Hay valued by Mensuration, must be far nearer the Truth, than that which is only guessed at by the most experienced Farmer. But I am sorry I have interrupted you with these impertinent Questions.

Philo. I am always ready to satisfy any doubt that may arise in your Mind; and will now proceed to the other Kind of Hay-ricks; that is, such as stand upon a rectangular Base.

The Figure A B C D E, placed in the Margin, represents a Hay-rick standing upon a rectangular Base; and is composed of two Solids: the upper A B C is a Prism; the lower A B D E, an irregular Solid having its Bases parallel but unequal, its two Sides equal but not parallel, and the Lengths, or distances, between its two Bases also unequal. The Content of the upper Solid may be found by the Rule given in Page 142, and the latter easily reduced to a parallelopiped, and the Content found by the Method laid down, Page 132.



EXAMPLE I.

A Farmer sells, at 35 Shillings a Load, a rectangular Rick of Hay of the following Dimensions; the greater Base A B is 36 Feet, the lesser Base D E 24 Feet; the greater Length, measured from A. and B, 80 feet, the lesser, or that measured from D and E 72 Feet; the Perpendicular Height of the lower Solid A B D E 20 Feet, and that of the upper A B C 28 Feet: What will the Whole amount to

1. Upper Solid.

$$\begin{array}{r}
 36 \text{ Base A B} \\
 14 \text{ Half Perp.} \\
 \hline
 144 \\
 36 \\
 \hline
 504 \\
 80 \text{ Length.} \\
 \hline
 40320 \text{ Content.}
 \end{array}$$

2 Lower

2. Lower Solid.

In order to find the content of the lower solid ABDE, you must find a mean base, by multiplying the two bases together, and extracting the square root out of the product; a mean length must also be found in the same manner. The solid will then be reduced to a parallelopiped, and the content found by the rule, page 132.

By pursuing this method the mean base will be 29.39, and the mean length 75.9. Then

$$\begin{array}{r}
 29.39 \text{ Mean Base.} \\
 \hline
 29.39 \\
 26451 \\
 817 \\
 \hline
 26451 \\
 5878 \\
 \hline
 863.7721 \\
 \hline
 75.9 \text{ Mean Length.} \\
 \hline
 77739489 \\
 43188605 \\
 \hline
 60464047 \\
 \hline
 65560.30239 \text{ Content.}
 \end{array}$$

$$\begin{array}{r}
 40320 \text{ Upper Solid.} \\
 65560.3, \text{ \&c. Lower.} \\
 \hline
 4.00(10588|0.3)264.7 \text{ Loads.} \\
 8... \\
 \hline
 25 \\
 24 \\
 \hline
 .18 \\
 16 \\
 .28 \\
 28 \\
 \hline
 .. \\
 \hline
 \end{array}$$

As

OF MEASURING.

309

As 1 Load : 1l. 15s. (=1.75) : : 264.7 :

$$\begin{array}{r}
 1.75 \\
 13235 \\
 18529 \\
 2647 \\
 \hline
 £463.225 \\
 20 \\
 \hline
 £. 4.500
 \end{array}$$

Anf. 463l. 4s. 6d.

EXAMPLE II.

A gentleman purchases of a farmer for 370l. a rectangular rick of hay of the following dimensions; the greater base 40 feet, the lesser 30; the greater length 70 feet, the lesser 60; the perpendicular height of the lower solid 20 feet, and of the upper 30; what did he pay per load?

1. Upper Solid.

$$\begin{array}{r}
 40 \text{ Base.} \\
 15 \text{ Half perp.} \\
 \hline
 200 \\
 40 \\
 \hline
 600 \\
 70 \text{ Length.} \\
 \hline
 42000
 \end{array}$$

R

2. For

2. For the lower solid.

34.64 Mean base.

34.64

13856

20784

13866

10392

1199.9296

64.8 Mean length.

95994368

47997184

7199577677755.43808 Content.

42000 Upper solid.

77755.438 Under.

4.00) 119755.4138 (299.388

8

39

36

37

36

15

12

35

32

34

32

2

Loads.

L.

L.

L.

d.

As 299.388 : 370 :: 1 : 2 :: 8

Ans. The content of the rick is 299.388 loads, and
the price per load 1*l.* 4*s.* 8*d.* $\frac{1}{2}$.

C H A P. IX.

Of measuring bodies, whose contents cannot be found by any of the foregoing rules.

Discip. **A**RE there any bodies whose solidity cannot be found by the common methods of mensuration?

Philo. Yes, several: as statues, and other pieces of sculpture, furze-faggots, &c.

Discip. By what method can the solid content of bodies of this kind be accurately known?

Philo. By a very easy method: it is this; let a vessel in the form of a parallelopiped be formed of sufficient capacity to contain the body, whose solidity is desired. Pour into this vessel as much water as will be sufficient to cover the body when placed in the vessel, and note the height of the water exactly: Immerse the body, and note how high the fluid has risen: for the solid content of the additional space, occupied by the water, on account of the immersed body, will be equal to the solidity of that body.

EXAMPLE I.

Suppose a parallelopiped to be formed, whose length is 7 feet, and its breadth 3 feet 6 inches; and that a statue being immersed in a sufficient quantity of water in this vessel, raises the fluid 1 foot 6 inches; what is the solid content of that statue?

$$\begin{array}{r}
 3.5 \text{ Breadth} \\
 \underline{7 \text{ Length.}} \\
 24.5 \\
 \underline{1.5 \text{ Rise of the water.}} \\
 1225 \\
 \underline{245} \\
 36.75 \text{ Solidity.}
 \end{array}$$

Ans. 36.75 feet, or 36 feet 9 inches.

EXAMPLE II.

Suppose a large furze-faggot was immersed in the same vessel, and the rise of the water was found to be 1 inch, what is the solid content?

$$\begin{array}{r}
 3.5 \text{ Breadth.} \\
 \underline{7 \text{ Length.}} \\
 24.5 \\
 \underline{.083 \text{ Rise of the water.}} \\
 735 \\
 \underline{1960} \\
 2.0335 \text{ Content.}
 \end{array}$$

CHAP. X.

A table of English measures, &c.

Philo. **I**T is not my intention to give you here the common table of English Measures; but such only as may be useful in Mensuration.

A Palm is 3 inches.

A Hand is 4 inches.

A Span is 9 inches.

A Square foot is 144 square inches.

A Cubic foot is 1728 cubic inches.

A Cubit is 1 foot 6 inches.

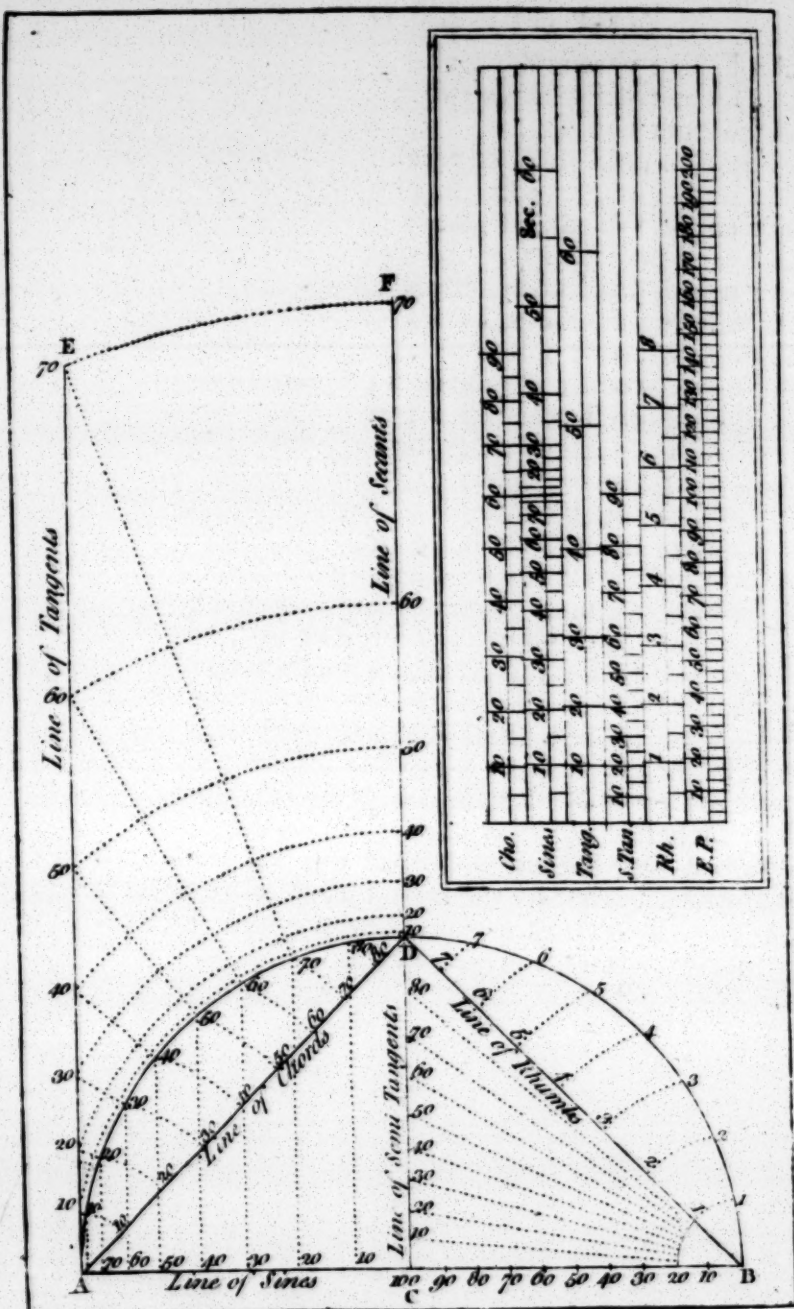
A Square yard is 9 square feet.

A Cubic yard is 27 cubic feet.

An

- An Ell is 1 yard and 9 inches.
 A Geometrical pace is 5 feet.
 A Fathom is 6 feet.
 A Statute pole, rod, or perch is 16 feet and a half.
 Gunter's chain is 4 statute poles, or 22 yards.
 A Fen, or Woodland pole is 18 feet.
 A Forest pole is 21 feet.
 A Furlong is 40 poles, or 220 yards.
 A Mile is 8 furlongs, or 1760 yards.
 A Square statute pole is 272.25 square feet.
 A Square woodland pole is 324 square feet.
 A Square forest pole is 441 feet.
 A Rood is 40 square poles, or 1 quarter of an acre.
 An acre 160 square poles.
 A Load of rough timber is 40 feet.
 A Load of squared timber is 50.
 A Load of 1 inch plank is 600 square feet.
 A Load of $1\frac{1}{2}$ inch plank is 400 square feet.
 A Load of 2 inch plank is 300 square feet.
 A Load of $2\frac{1}{2}$ inch plank is 240 square feet.
 A Load of 3 inch plank is 200 square feet.
 A Load of $3\frac{1}{2}$ inch plank is 170 square feet.
 A Load of 4 inch plank is 150 feet square.
 A Chaldron of Newcastle coal is 60 cubic feet.
 A Winchester bushel is 2150.4 solid inches.

When the bushel is heaped up, the content is in proportion to that of the same bushel when struck as 4 to 3. Therefore the conical heap is one third of the cylindrical content, or 716.8 solid inches.



C H A P. XI.

Of the plain scale, the manner of constructing the several lines upon it, and the uses it is generally applied to in the various branches of mathematical computation.

Discip. **W**HAT do you mean by a plain scale?

Philo. A plain scale, is a thin piece of brass, ivory, or wood, generally about 12 inches long, and 1 inch and a half broad; on which various lines are projected, for the mensuration of angles, heights, distances, &c.

Discip. Why is it called a plain scale?

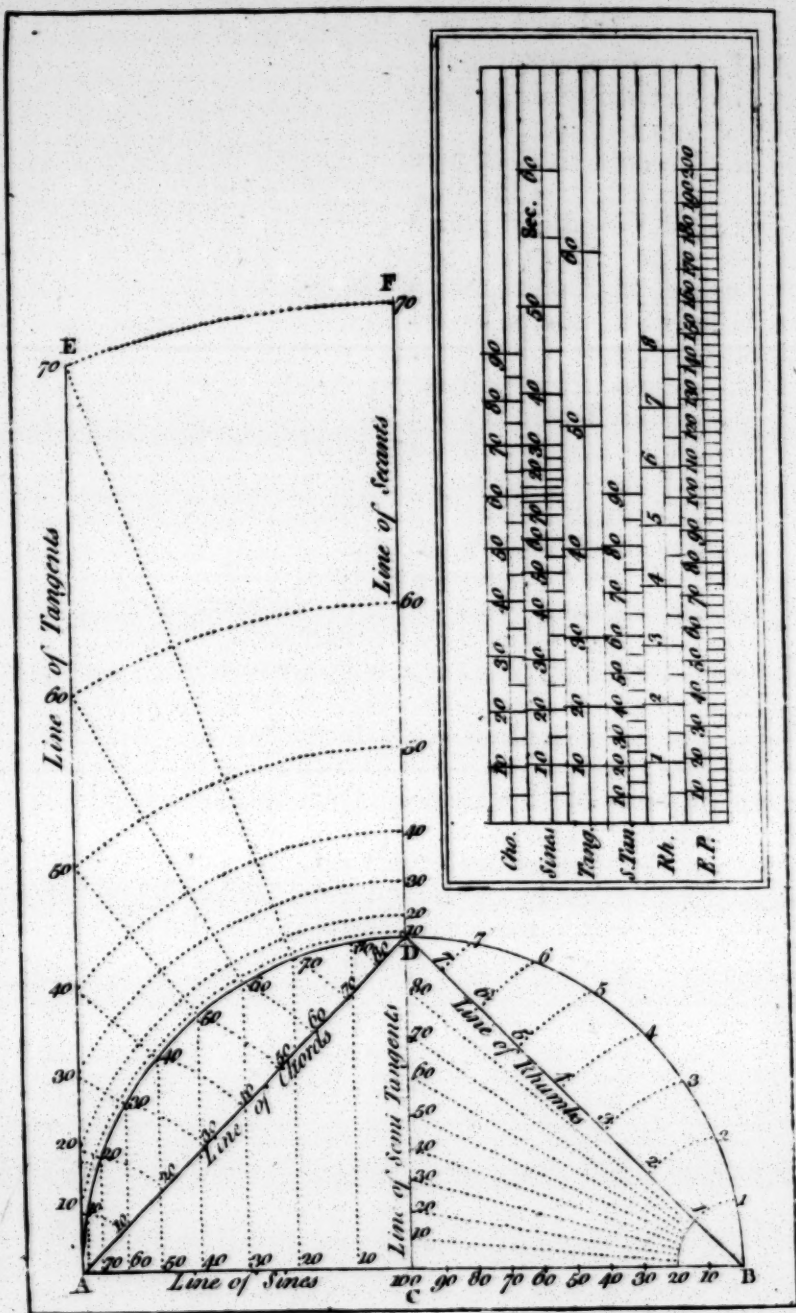
Philo. Because the lines projected upon it are such as naturally result from a circle, without having recourse to any artificial numbers, &c.

Discip. What are the lines generally placed on a plain scale?

Philo. A great variety of lines may be projected for this purpose, and all of them have their uses; but the most general are, a line of chords, of sines, of tangents, of secants, semi-tangents, and equal parts.

Discip. How are those lines projected?

Philo. They are all graduated from the circle, in a very natural manner. But in order to render this useful part of mathematical construction as plain and easy as possible, I shall consider each line separately, and shew how each of them may be projected, and consequently, the plain scale completely finished by the meanest capacity. But it will be necessary previously to observe, that the circumference of every circle is supposed to be divided into 360 equal parts, called degrees; and each degree into 60 equal parts called minutes; and each minute into 60 equal parts called seconds, &c.



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1. *Of the line of chords.*

The chord, or subtense of an arch, is a right line connecting the extremities of the arch together, and has been already defined, page 31. If, therefore, the several degrees of the quadrant, or quarter of a circle, be transferred to a right-line, drawn from one extremity of the quadrant to the other, that line will be a line of chords.

In order to explain this description of a line of chords, place one foot of the compasses in the center C (see the figure on page 314.) and with any convenient radius, describe the semi-circle ADB, and divide it into two equal parts or quadrants, by the line CD. Divide the arch of the quadrant into 9 equal parts, and number these divisions with 10, 20, 30, 40, &c. from A to D, as you see in the figure. Then will each of these divisions contain 10 degrees: subdivide each of these divisions into 10 equal parts; so will the whole quadrant be divided into 90 degrees. But as these subdivisions would have been so very close together in the figure annexed, as to have rendered the whole confused, they are omitted. The arch being thus divided draw the right-line AD, from A, the one extremity of the arch, to D the other. Place one foot of the compasses in A, and extend the other to 10 on the arch, and transfer that distance to 10 on the right-line: that is, keep one of the points of the compasses firm on the point A, and with the other sweep a small arch, till it touches the right-line AD. Do the same by all the rest of the divisions, 20, 30, 40, &c. Keeping one of the points still fixed on the point A. By this means the equal divisions of the arch will be transferred to the right-line, where they will become unequal; and the line AD divided in this manner, will be a line of chords.

You must not imagine that a line of chords projected from a circle of so small a radius can be of any real use in practice: The radius of the circle must be much larger,

larger, that there may be a proper space between the divisions on the line when finished; otherwise it will be impossible to measure or construct any angle with sufficient accuracy. The foregoing figure was added merely to explain the manner of constructing the several lines on the plain scale; and not by any means to serve as an example with regard to its magnitude.

2. *Of the line of sines.*

From the points of the several divisions 10, 20, 30, &c. of the quadrantal arch AD, draw lines parallel to DC, till they cut the radius AC, then will the right-line CA be a line of sines. And if we suppose the arch AD, to be divided into 90 equal parts, and that lines parallel to DC be drawn from the several points of those divisions, the line CA will be divided into a line of sines, answering to every single degree of the quadrant, and which must be numbered from C towards A.

N. B. If the line AC, divided as before, be numbered from A towards C, you will have a line of versed sines, defined page 31.

3. *Of the line of Tangents.*

From A, the extreme point of the arch, raise the perpendicular AE; and from the center C, through the several divisions of the quadrant AD, draw lines till they cut the tangent AE (see page 31.) so then will the line AE, become a line of tangents, and must be numbered from A towards E. But as the distances between the divisions become very large, as they approach towards 90 degrees, the tangent line is continued no farther than 70 degrees, in order to keep it within a convenient length.

4. *Of the line of Secants.*

The secant of any arch has been already defined, page 31. and consequently the lines drawn from the center C, through the several divisions of the quadrant AD, to divide the line of tangents, are the secants of their respective arches: If therefore, their lengths be transferred to the line CD, continued to F, the line CF will be a line of secants; and must be numbered from C towards F.

After the same manner may the lines of sines, tangents, and secants, be constructed to every minute of the quadrant, provided the radius be large enough to admit of the arches being divided into the requisite number of equal parts.

5. *Of the line of Semi-Tangents.*

The line of semi-tangents is so called from the lengths of the several divisions, being equal to the respective tangents of half that number of degrees. That is, the length of the semi-tangent of 90 degrees, is equal to the tangent of 45 degrees; the length of the semi-tangent of 80 is equal to the tangent of 40, &c. This being premised, the line is easily projected in the following manner: lay a ruler from the point B, to the several divisions of the quadrant AD, and mark where it cuts the line CD; for these divisions on CD will be the semi-tangents of their respective arches; and consequently the line CD will be a line of semi-tangents, and must be numbered from C towards D.

6. *Of the line of Rhumbs, or Rumbs.*

By the term rhumbs are meant the points of the Mariner's Compass, which is divided into 32 equal parts: consequently each quadrant of the circle contains eight of those points or rhumbs; and the interval between them is equal to 11 degrees, 15 minutes; for

11 degrees, 15 minutes, multiplied by 8 is equal to 90, the number of degrees contained in a quadrant of a circle. The line of rhumbs is therefore projected in the following manner:

Divide the quadrantal arch BD into 8 equal parts; and draw the right-line BD, from one extremity of the arch to the other. This being done, transfer the several divisions of the arch to the right-line: That is, place one foot of the compasses in the point B, and extend the other successively to the several divisions of the quadrant, and with these extents, draw small arches till the other foot touches the line BD: then will that line thus divided be a line of rhumbs, which must be numbered from B towards D.

Discip. You mentioned the mariner's compass in explaining the method of constructing the line of rhumbs; I should be glad if you would give me some account of that instrument, because otherwise I shall know very little of this line.

Philo. The mariner's compass is the instrument by which the ship is steered, and represents the horizon. It is divided into 32 equal parts called rhumbs or points, by lines drawn from the center to the circumference; and the outer circle into 360 degrees. Upon this part, and directly on the north and south line, is fastened a square bar of steel, which, being touched with a loadstone, acquires a certain virtue, whereby it always hangs nearly under the meridian, when the whole is suspended on the point of a needle, placed perpendicularly under the center in a box for that purpose.

7. *Of the line of equal Parts.*

This line is easily divided when it is determined how many of these parts the radius is to contain. If the scale be intended for the purposes of navigation, it should be divided into 60 equal parts; because 60 geographical miles makes a degree of longitude on the equator: and as the length of a degree of longitude in any parallel, is to the length of a degree of the equator,

as

as the co-sine of the latitude is to radius; therefore the length of a degree on any parallel may be easily measured by it. But as this scale is not confined to navigation, I have divided the radius, or C B, into 100 equal parts.

Use of the plain Scale.

This scale is of great use in every branch of the mathematics, where there is a necessity for laying down and measuring angles, &c.

I have shewn the method of graduating a plain and diagonal scale of equal parts, page 50, 51, &c. and their uses in surveying land; and shall now shew the use of the scale just described, in projecting and measuring any of the parts of a triangle, when a sufficient number of these parts are given.

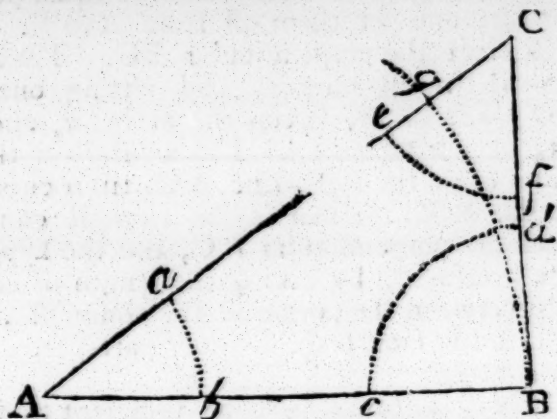
For it is necessary, before any question can be solved, that proper data be given, or found by actual mensuration; and these data are what I called the given parts of a triangle. It is impossible, for instance, to find the area of a square, unless the side, or something analogous to it, be given; the same may be said of a triangle, or any other geometrical figure.

Discip. You have fully explained the difficulty; and I beg you will now proceed to the mensuration of the several parts of a triangle.

Phil. A triangle is composed of three sides and three angles; when one of the angles is just 90 degrees, it is called a right-angled triangle; but if neither of the angles be a right one, it is called an oblique-angled triangle.

23 OC 62

The figure below is a right-angled triangle; because the angle at B, or ABC , which is measured by the arch cd , is equal to 90 degrees. The line AB is called



the base, BC the perpendicular, and AC the hypotenuse. The angle at A is measured by the arch ab , and that at C by the arch ef .

It is demonstrated by geometrical writers, that the three angles of any right-lined triangle are equal to 180 degrees. But one of the angles of a right-angled triangle is equal to 90 degrees; therefore the sum of the other two angles is equal to 90 degrees; consequently if one of these angles be given, the other will be easily found, by subtracting the given angle from 90 degrees.

This being premised, it will be easy to project and measure any of the parts of a right-angled triangle, if either the base, perpendicular, or hypotenuse, together with one of the oblique angles be given.

EXAMPLE.

Suppose the base AB , be 80 yards, and the angle at A 35 deg. and let it be required to project the angle BAC , and measure the perpendicular BC , and the hypotenuse AC .

S

SOLU-

SOLUTION.

Draw the right-line AB ; take 80 equal parts from your scale, and set them off from A to B . On the point B erect the perpendicular BC . Take 60 deg. from your line of chords, and setting one foot of the compasses in A , sweep the arch ba , and set off on it 35 deg. from b to a . Lay a ruler on the points A, a , and draw the right-line AC , till it cuts the perpendicular in C . Then is the triangle constructed, and both the perpendicular BC , and the hypotenuse AC may be measured, by taking the length of each successively between the points of the compasses, and applying it to the same scale of equal parts, from whence AB was taken.

The above Solution will be sufficient to shew the method of constructing a triangle, and measuring its several parts. It is also evident from the figure that if AB be considered as the radius, and the arch Bg be drawn, that BC will be the tangent of the angle at A and AC its secant. If CB be supposed to be the radius, BA will be the tangent of the angle at C and CA its secant. But if AC be supposed the radius, then will AB be the sine of the angle at C , and CB , the sine of the angle at A .

These few observations will be sufficient to point out the uses of the plain scale, in the doctrine of triangles, on which most of the mathematical sciences depend.

If BC represent a tower, tree, &c. and you measure the base AB , and take the angle at A , with some proper instrument, the triangle may be easily projected, and the height of the object measured on the scale of equal parts, from whence AB was taken. Some examples of this are given in page 97, 98, &c.

Thus have I conducted you through every part of what is generally called mensuration; and shall now leave you to put these precepts in practice. On some future occasion, perhaps, I may instruct you in other branches of the mathematics, in the same easy and familiar manner.

25 OCT 62
F I N I S.

